Total Wealth, Consumption and Portfolio Shares: Evidence and Theory*

Michel Normandin
Department of Economics, and CREFE
Université du Québec à Montréal

Pascal St-Amour
Department of Finance
Hautes Études Commerciales, Montréal

March 12, 2001

*Corresponding author: Pascal St-Amour, Department of Finance, Hautes Études Commerciales, 3000, ch. de la Cote-Ste-Catherine, Montreal, Qc, Canada, H3T-2A7, tel. (514) 340-6556, fax (514) 340-5632, e-mail: Pascal.St-Amour@hec.ca.
Normandin acknowledges financial support from FCAR. St-Amour is grateful for support by SSHRC. We have benefited from useful comments and suggestions made by Stephen Gordon, Angelo Melino, and Simon Van Norden. The usual disclaimer applies.

Abstract

This paper documents the empirical aggregate consumption and portfolio allocations without imposing severe assumptions to construct total wealth and its return. This results in realistic new measures of these components, which in turn, yield highly plausible implicit asset returns, but smaller consumption and portfolio shares than conventional educated guesses. This analysis also derives the theoretical consumption and portfolio allocations predicted by flexible specifications of both investors' preferences and investment opportunities. This leads to a predicted share for stock that is small compared to that obtained in earlier work, and relative to the other shares that we consider. Finally, this study statistically confronts, for one of the first time, the empirical allocations to their theoretical counterparts. This exercise confirms many puzzles documented in the asset returns' literature and highlights other important anomalies, which are actually even more pronounced for assets other than stock.

Keywords: Elasticity of intertemporal substitution; dynamic hedging; risk aversion; time-varying investment opportunity set.

JEL classification: G11

Introduction

The classical dynamic asset pricing paradigm stipulates that investors jointly determine consumption and portfolio decisions. A popular approach is to study the underlying pricing implications, given an exogenous stochastic process for equilibrium consumption. The dual is to directly gauge the underlying consumption and portfolio rules, taking as given an exogenous stochastic process for excess returns.

This paper is related to the second line of research. More precisely, we pursue three objectives. The first goal is to document the empirical aggregate consumption and portfolio allocations using an alternative approach, which relaxes the severe assumptions required to construct total wealth and its return. This results in realistic new measures of these components, which in turn, yield highly plausible implicit asset returns, but smaller consumption and portfolio shares than conventional educated guesses. The second aim is to study the theoretical consumption and portfolio allocations predicted by a generalized setting, which involves flexible specifications of both the investors' preferences and the temporal evolution of investment opportunities. This leads to a predicted share for stock that is small compared to that obtained in earlier work, and relative to the other shares that we consider. The final objective is to confront, for one of the first time, the empirical allocations to their theoretical counterparts by conducting several formal statistical tests. This exercise confirms many puzzles documented in the asset returns' literature and highlights other important anomalies, which are actually even more pronounced for assets other than stock.

First, the description of empirical allocations mainly relies on the measurement of consumption and portfolio shares relative to total wealth. Conceptually, this wealth corresponds to the sum of tangible, financial, and human assets. Unfortunately, both human wealth and its associated return are not observable. One approach measures human wealth from its replacement cost, incorporating elements such as child-rearing and education costs (Kendrick 1976, Eisner 1989). Another method evaluates human wealth from the expected net present value of the flow of revenues accruing to the owners of human capital, which is generally assumed to correspond to labor incomes. This alternative imposes strong restrictions on movements in returns and/or time-series properties of income. For example, Chou, Engle and Kane (1992) assume that the returns on stock and portfolio are always identical, while Campbell (1996) imposes that the returns on human and financial assets are the same on average, whereas Jorgenson and Fraumeni (1989) and Shiller

(1995) postulate that the rate discounting future income is constant. Also, Fama and Schwert (1977) and Jagannathan and Wang (1993) impose that labor-income growth is unforecastable, while Heaton and Lucas (1997) and (2000a) assume that each agent's labor income remains constant at its current level until age 65, and then ceases.

Here, we develop a different approach which simply exploits the investors' intertemporal budget constraint and the induced national saving identity. More specifically, we evaluate these equations from historical series for national income and aggregate consumption to recursively construct total wealth and its implicit return. We also resort to official aggregate data for some selected asset values and the constructed total wealth to compute consumption and portfolio shares. Given these shares, it is possible to calculate the implicit excess return associated with each asset of interest, again through the budget constraint. This is particularly useful to the extent that some of these assets have no published returns' index series.

Our sample consists of postwar annual US data. The consumption series corresponds to aggregate expenditures on nondurables, and selected assets are based on the Flow of Funds stock series on deposits, pension and insurance reserves, as well as corporate, noncorporate, and home equities. Our measures for total wealth and its associated return are reasonable. In particular, our total wealth series lies between those obtained using the replacement cost and net present value methods. Also, our return on total wealth accords with the well-accepted notion that the portfolio displays a much stronger degree of diversification than stock-market indices. Using our constructed total wealth series, we find that all the average shares are smaller than those usually reported in previous studies, while the share for corporate equity displays atypically large volatility and small covolatility. These stylized facts indicate that the standard practice of focusing mainly on stocks provides an incomplete characterization of the actual investors' portfolio. We also obtain that the empirical portfolio mix is consistent with popular financial advisors' recommendations: the ratio of other assets to stock holdings should increase in risk aversion. Next, the average excess returns are all positive, but are substantially larger for reserves and corporate equity. The excess returns on corporate equity is by far the most volatile and is the only one that covaries negatively with the excess return on the total wealth portfolio. These empirical regularities illustrate Roll's critique stating that stock-market indices are inadequate proxies for the portfolio return. In addition, we show that the implicit excess returns on corporate and home equities are remarkably similar to the actual excess returns on the S&P500 stock-market index and on housing-market indices. This evidence suggests that our constructed series are accurately measured. The stylized facts documenting the empirical consumption and portfolio allocations are detailed in Section 1.

Second, the theoretical allocations are studied through a representative investor characterized by time- and state-nonseparable preferences and facing a time-varying investment opportunity set. This environment is designed to relax the conditions required to yield constant shares (Samuelson 1969, Merton 1971). As illustrated by the pronounced fluctuations in the aggregate value of corporate stocks relative to consumption expenditures in Figure 1, such a prediction is clearly at odds with the data. Also, our environment is useful since it allows the disentanglement of the investors' attitudes towards risk and intertemporal substitution on their optimal consumption and portfolio shares.

In general, our specification of the investor's problem has no exact analytical solution. We circumvent this problem by deriving an analytical approximation of the consumption and portfolio decision rules. Our approximation is related to recent analyses characterizing the optimal decision rules of an investor having nonseparable preferences (Campbell and Viceira 1999, Campbell, Chan and Viceira 1999). However, we depart from these studies by describing changes in investment opportunities through a multivariate stochastic process relating contemporaneous excess returns to lagged values of a single state variable. This state is governed by an autoregressive process, and is treated as a latent variable. The latent-variable approach is motivated by the absence of evidence establishing which (if any) published series jointly Granger-cause all our selected individual excess returns. Importantly, our multivariate process constitutes a richer alternative to the univariate specification used in Campbell and Viceira (1999), where the single risky excess return on corporate equity is linked to past values of the associated dividend-price ratio, which also follows an autoregressive process. Moreover, our specification offers the advantage of being more parsimonious than the vector autoregressive process employed in Campbell et al. (1999), which captures the interactions of several excess returns and other observable state variables, including stock-index dividend yields.

Our analytical approximation reveals that the consumption share decreases in the elasticity of intertemporal substitution at low values of relative risk aversion, while it increases at high degrees of aversion to risk. We explain this complex pattern from the intertemporal substitution and income effects associated with changes in investment opportunities. In addition, we find that the portfolio shares are almost completely insensitive to the elasticity of intertemporal substitution, but rapidly decline in relative risk aversion. We rationalize this behavior from the disentanglement of each asset total demand into myopic and hedging components. We show that both components for corporate equity are smaller than those obtained in previous work; this mainly arises because our latent variable exhibits different time-series properties than those of the dividend-price ratio. We also demonstrate that these components are substantially smaller than those predicted for the other risky assets; this occurs because stocks display the smallest Sharpe ratio, and they produce a return which provides the weakest signal about future return movements. This implies that the predicted portfolio mix is such that the ratio of other assets to corporate equity holdings decreases in risk aversion. The predictions characterizing the theoretical consumption and portfolio allocations are reported in Section 2.

Finally, the comparison between empirical and theoretical allocations is achieved by conducting formal statistical tests. These tests verify whether the average actual and predicted shares are identical. These tests also check whether the estimates of unrestricted reduced forms and the predicted coefficients of the corresponding optimal decision rules are the same. All the tests take into account the uncertainty related to the estimates of our multivariate stochastic process for excess returns. This study innovates by performing a formal empirical evaluation of a generalized representative-agent model of optimal consumption and portfolio shares at the aggregate level.

Our test results indicate that the average actual shares are almost always statistically over-predicted. At best, we obtain that the means of empirical shares for consumption, deposits, reserves, and corporate equity can only be matched under extremely low elasticities of intertemporal substitution and excessively high relative risk aversion. For these parameter values, we find that the best results are obtained when the elasticity of intertemporal substitution is the reciprocal of relative risk aversion, so that we cannot refute the validity of separable preferences. This accords with previous simulation and test results obtained by evaluating the underlying asset pricing implications (Weil 1989, Jorion and Giovannini 1993, Normandin and St-Amour 1998). This finding suggests that the observed consumption and portfolio shares cannot be explained by the nonseparability of preferences. Furthermore, the investor's strong reluctance to substitute consumption across periods is consistent with the fact that the historical aggregate consumption is smooth (Deaton 1987, Campbell and Deaton 1989), is required to reconcile changes in this series to observed fluctuations in real interest rate (Hall 1988, Campbell and Mankiw 1989), and is directly related to the riskfree rate puzzle (Weil 1989). Likewise, the investors' strong distaste for risk is closely linked to the equity premium and stockholding puzzles, where an unreasonably high relative risk aversion is necessary to reconcile average excess return on stock to its low quantity of risk and to the small proportion of the population holding this asset directly (Mehra and Prescott 1985, Mankiw and Zeldes 1991, Haliassos and Bertaut 1995). Finally, our results suggest that the predicted portfolio mix is at odds with its empirical counterpart. This confirms that the asset allocation puzzle also holds in the aggregate, but does not support the conjecture by Canner, Mankiw and Weil (1997) that richer preferences and environment may solve the anomaly.

Importantly, our results also highlight several other anomalies not documented so far in the literature. In particular, we show that our analogue of the stockholding puzzle is not specific to corporate equity, but prevails for all assets: the low actual portfolio shares are always significantly over-predicted using plausible values of relative risk aversion. This anomaly is in fact more severe for noncorporate and home equities, given that the associated empirical shares are statistically over-evaluated even under extremely high relative risk aversion. Moreover, we find that the estimates of the relevant unrestricted reduced forms are most of the time significantly different from the predicted coefficients of the consumption and portfolio decision rules. This implies important anomalies regarding the fundamental decomposition of intertemporal substitution and income effects related to changes in investment opportunities as well as the fragmentation of asset total demands into myopic and hedging behaviors. The test results confronting the empirical consumption and portfolio allocations to their theoretical counterparts are presented in Section 3.

1 Empirical Consumption and Portfolio Allocations

This section presents the US stylized facts on the value and the rate of return on total wealth. We subsequently document consumption and portfolio shares, followed by individual returns.

1.1 Value and Rate of Return on Total Wealth

We first construct empirical time series of total wealth and its associated return. This exercise is performed by using two equations. The first equation is the investors' intertemporal budget constraint. This constraint states that the next-period wealth corresponds to the wealth portfolio gross return times the amount of contemporary wealth not used for consumption:

$$W_{t+1} = (1 + r_{p,t+1})(W_t - C_t), \tag{1}$$

where W_t is aggregate total wealth in period t, C_t is aggregate private consumption, $r_{p,t+1} = \boldsymbol{\alpha}_t' \boldsymbol{r}_{t+1}$ is the (net) return on the total wealth portfolio, $\boldsymbol{r}_{t+1}$ is the N-vector of returns on individual assets, and $\boldsymbol{\alpha}_t$ is the N-vector of portfolio shares – so that $\boldsymbol{\alpha}_t' \boldsymbol{\iota} = 1$ and $\boldsymbol{\iota}$ is a unit vector. Note that total wealth includes both human and nonhuman wealth. Likewise, the portfolio return takes into account the human-wealth and nonhuman-wealth returns. Importantly, the human wealth and its associated return do not explicitly involve labor income.

The second equation is the income-wealth identity. This identity is obtained by defining national income as the income from all productive assets:

$$Y_t \equiv \frac{r_{p,t+1}}{1 + r_{p,t+1}} W_t, \tag{2}$$

where Y_t is national income. Chou et al. (1992) implicitly consider a variant of (2) by using $Y_t = r_{p,t+1} W_t$. Furthermore, Heaton and Lucas (2000a) use a similar method to infer the stock of wealth invested in bonds (interest income divided by the rate of return on T-Bills) and corporate equity (dividend income divided by the dividend yield on the S&P500).

Equation (2) can be rationalized by rewriting the budget constraint (1) as $(W_{t+1} - W_t)/(1 + r_{p,t+1}) = [r_{p,t+1}/(1+r_{p,t+1})]W_t - C_t$, which is precisely the fundamental private-economy saving identity: $S_t \equiv Y_t - C_t$, where S_t is aggregate saving. In the spirit of Jagannathan and Wang (1996), an alternative interpretation of (2) is obtained from the present value $W_t = \mathrm{E}_t \sum_{i=0}^{\infty} \prod_{j=0}^{i} z_{t+j} Y_{t+i} = Y_t \mathrm{E}_t \sum_{i=0}^{\infty} \prod_{j=0}^{i} z_{t+j} g_{y,t+j}$, such that $(1 + r_{p,t+1})/r_{p,t+1} = \mathrm{E}_t \sum_{i=0}^{\infty} \prod_{j=0}^{i} z_{t+j} g_{y,t+j}$ – where $g_{y,t}$ is the gross growth rate of income and z_t is a stochastic discount factor. Hence, our portfolio return is consistent with both time-varying income growth and discount rates. However, our approach has the advantage of not imposing any restrictions on these rates.

The budget constraint (1) and the income-wealth identity (2) are used to construct the portfolio return and total wealth. The portfolio return $r_{p,t+1} = Y_t/(W_t - Y_t)$, induced by (2), is measured from historical national income and the constructed total wealth. Also, total wealth $W_{t+1} = (1 + r_{p,t+1})(W_t - C_t)$, obtained from (1), is computed from actual consumption and the constructed portfolio return. This recursive construction of $r_{p,t+1}$ and W_t for the time periods $t = 2, \ldots, T$ is performed for initial values $r_{p,2}$ and W_1. The term $r_{p,2} = (g_y - 1)/(1 - g_c)$ is fixed to the steady state of the portfolio return – where g_w, g_c, and g_y are steady states obtained from the stationary-inducing transformation for growing variables: $g_{w,t} = W_t/Y_t = (1 + r_{p,t+1})/r_{p,t+1}$, $g_{c,t} = C_t/Y_t$, and $g_{y,t} = Y_t/Y_{t-1}$. The term $W_1 = (1 + r_{p,2})Y_1/r_{p,2}$ is computed from the steady-state value of the portfolio return and the initial observation of historical national income.

Note that official measures of national income exclude capital gains, such that they are not included in our constructed measure of total wealth. Heaton and Lucas (2000a) argue that realized capital gains are poor indicators of the true value of asset stocks. In fact, it should be kept in mind that realized capital gains are marginal compared to total revenues. For instance, in 1999, the net gains from the sale of capital assets, for a median household were a meager \$ 552, out of annual money income of over \$ 40,000 (source: US Census Bureau, Current Population Survey, March 1999 and 2000).

The historical series for national income and consumption are US annual real, per-capita, gross domestic product and private aggregate expenditures on nondurable goods and services for the 1945-1998 period. These series are obtained by normalizing the nominal data (source: National Income and Product Accounts) by the gross domestic product implicit deflator (base year 1992) and by the total population (source: Census Bureau). Note that the series are expressed in real, per-capita, term to facilitate eventual comparisons between the stylized facts and the predictions, obtained from a real-economy, representative-investor model. Also, the sample period is chosen to match available official series for asset values of interest, as discussed below. Furthermore, the annual frequency is selected for a number of reasons that will prove to be useful for our analysis. First, only annual values are recorded for some relevant assets. Second, annual returns purge seasonal return movements (if any) and this will make easier to compare the implicit individual returns constructed from the historical gross domestic product — which is only available on a seasonally-adjusted basis — to official index returns — which are reported on a seasonally-unadjusted basis. Third, annual returns are likely to be conditionally homoscedastic, a feature that will simplify the derivation of the optimal consumption and portfolio rules predicted by the model.

Empirically, the ratio of the constructed total wealth to actual national income has a mean of 30.26, a standard deviation of 2.60, and has extremes of 27.14 and 36.24. These stylized facts reveal that wealth is substantially larger than income. To get an idea on the scale of these variables, real, per-capita, total wealth was $ 762,359 in 1998, while real, per-capita, gross domestic product in 1998 was $ 27,939. In addition, the portfolio excess return (where the riskfree rate is proxied by the real return on three-month US T-Bills) exhibits a mean of 2.66%, a standard deviation of 3.15%, and ranges from -2.24% to 15.18%. Hence, the excess return on total wealth is reasonable, on average, and is fairly smooth. Below, we will present a complete comparison between the portfolio excess return and several individual excess returns. For the moment, note that the mean return on total wealth lies between the average stock-index return and the average short-term real interest rate.

The validity of our constructed series just presented is verified by comparing them to those obtained from alternative conceptually different methods, namely the replacement cost and present value approaches. The first approach evaluates the costs required to replace nonhuman and human stocks. For example, tangible nonhuman stock is assessed from the costs of land and structures, while intangible nonhuman assets are valued from expenditures devoted to research and developments. Also, tangible human stock is captured by cumulated child rearing costs, whereas the intangible human wealth is measured from the costs associated with education, training, health, and mobility (job search, frictional unemployment, and migration). The second approach evaluates total wealth from the present values of expected future net incomes associated with the various assets. Incomes related to human wealth can be either restricted to labor income or can also include the value of time allocated to nonmarket activities.

Table 1 confronts summary statistics for the wealth-income ratio and the portfolio excess return. This ratio is computed from real, per capita, actual gross domestic product and several series of total wealth constructed from the methods described above. The return on total wealth is calculated from the budget constraint (1) and real, per capita, observed income, historical consumption expenditures, and various constructed total wealth series. As already explained, the total wealth W_{bc} generated from our budget constraint procedure is constructed recursively from (1) and (2). The total wealth, W_{rc}, for the replacement cost approach is provided by Eisner (1989). This annual series is constructed in a similar way than Kendrick's (1976) classical measure, but it covers a longer period. Also, we compute a first total wealth series, $W_{pv,1}$, for the present value method. For this purpose, we add the value of tangible and financial assets (source: Balance Sheet of Households and Nonprofit Organizations) to human wealth — where this last component is evaluated by constructing expected future labor incomes from a first-order autoregressive process for total compensation of employees (source: National Income and Product Accounts) and by using a deterministic discount rate of 3.5%. Finally, a second total wealth series, $W_{pv,2}$, for the present value method is taken from Jorgenson and Fraumeni (1989). This annual series also values the nonmarket activities, such as volunteer work, commuting, and leisure. The series W_{bc}, W_{rc}, $W_{pv,1}$, and $W_{pv,2}$ all cover the 1949-1981 common period.

The comparison exercise for the common subsample leads to two observations. First, the wealth-income ratio reveals that our method yields a measure of total wealth W_{bc} which is, on average, larger than W_{rc}, similar to $W_{pv,1}$, and smaller than $W_{pv,2}$. For example, in 1981 real, per-capita, total wealth was evaluated at \$ 156,792 for W_{rc}, \$ 587,166 for W_{bc}, \$ 674,518 for $W_{pv,1}$, and \$ 1,105,667 for $W_{pv,2}$, while real, per-capita, national income was \$ 20,571. Second, the portfolio excess return computed from our measure W_{bc} is on average smaller than that obtained from W_{rc}, and is similar to those induced by $W_{pv,1}$ and $W_{pv,2}$. Note that these findings are presumably robust to the choice of the sample, given that for W_{bc} the descriptive statistics for the 1949-1981 period are very similar to those already presented for the 1945-1998 period.

Clearly, our total wealth series W_{bc} is not an outlier: it yields a mean wealth-income ratio that is very close to the midpoint. Moreover, the excess return associated with our total wealth accords with the conventional wisdom that the portfolio displays a much stronger degree of diversification than corporate equity. This translates into a smaller and smoother portfolio excess return than the compounded annual excess return on the S&P500 stock-market index, which displays for the 1949-1981 period a mean of 7.92%, a standard deviation of 16.57%, and extremes of -26.81% and 44.87%.

Furthermore, additional elements lead us to assert that our measures are realistic. First, the portfolio excess return induced by the total wealth series W_{rc} exhibits unreasonable movements; it is substantially larger on average, but much smoother than excess return on corporate equity. Secondly, our constructed portfolio return is similar to the one obtained for $W_{pv,2}$, which includes nonmarketable activities in the computation of human wealth. Abstracting for these activities, as in $W_{pv,1}$, yields almost identical measures to ours for both total wealth and its rate of return. Unfortunately, the evaluation of nonmarket activities remains a subjective issue, and whether activities such as volunteer work, commuting and leisure are true contributions to human wealth is debatable. Finally, our constructed time-series for total wealth is consistent with measures found in cross-sectional analysis. For example, Heaton and Lucas (2000a) obtain an average of $ 1,000,000 per household in 1992, or $ 417,688 per capita, based on the 1990 Census estimates of 2.63 individuals per median household. For that year, our estimate is $ 698,004. The difference can be explained from their assumptions that the real labor income is constant until age 65, and then ceases, and that this income stream is discounted at a relatively high rate of 5%. Allowing for income growth, a longer horizon or a lower discount rate – as we do in computing $W_{pv,1}$ – increases the value of total wealth.

We believe that these elements convincingly show that our constructed series of total wealth and its associated rate of return are correctly measured. This conclusion will be further supported by additional evidence reported below.

1.2 Consumption and Portfolio Shares

We next construct empirical time series for ratios of consumption and portfolio relative to total wealth. For this purpose, we use the definitions:

$$\alpha_{c,t} \equiv \frac{C_t}{W_t}, \tag{3}$$

and,

$$\alpha_{i,t} \equiv \frac{W_{i,t}}{W_t}, \tag{4}$$

where $\alpha_{c,t}$ represents the consumption share, while $\alpha_{i,t}$ and $W_{i,t}$ are the portfolio share and the aggregate value of asset i, respectively.

In principle, we could conduct the analysis of the shares associated with all the N individual assets composing the portfolio but, we limit attention to a subset of investors' assets $i = 1, \ldots, n$, where $n < N$, for two reasons. First, the empirical shares for some assets cannot be evaluated because their historical values are not published. Second, as will become clear later on, evaluating the shares predicted by the model requires the estimation of a multivariate process for individual returns, and including any additional assets reduces considerably the degrees of freedom.

The constructed consumption share, obtained from (3), is computed for $t = 1, \ldots, T$ from the real, per-capita, actual nondurables and services consumption and constructed total wealth. The portfolio shares, (4), are constructed by also using real, per-capita, actual values for selected assets. These real value series are obtained by normalizing each asset aggregate nominal value by the gross domestic product implicit deflator and the total population. The nominal values are taken from the "Balance Sheet of Households and Nonprofit Organizations", published by the Board of Governors of the Federal Reserve Bank. The selected nominal values are deposits (mnemonic FL154000005), reserves (pension fund reserves, FL153050005, plus life insurance reserves, FL153040005, net of deferred and unpaid insurance premiums, FL543077003), corporate equity (FL153064105), noncorporate equity (FL153080015), and home equity (real estate, FL155035015, net of home mortgages, FL153165105). These series cover the 1945–1998 period. The availability of the data for the value of home equity limits our analysis to the annual frequency.

Our selection of the specific assets is guided by a number of criteria. First, corporate equity provides a very useful benchmark, since the behavior related to this asset has been carefully documented in previous work. Second, the selected assets cover two major classes of nonhuman wealth: financial wealth (deposits, reserves, as well as corporate and noncorporate equities) and tangible wealth (home equity). Third, these assets represent substantial portions of financial and tangible wealth. For example, the values of deposits, reserves, corporate equity, and noncorporate equity correspond, on average, to 20%, 17%, 19%, and 28% of financial wealth. Again, to illustrate the value of these figures, the real, per-capita, values of these assets were \$ 13,638, \$ 30,940, \$ 20,681, and \$ 14,165 in 1998; while the real, per-capita, financial wealth was

$ 99,350. Also, the value of home equity represents, on average, 47% of tangible wealth. In 1998, the real, per-capita, value for home equity was $ 17,030; while real, per-capita, tangible wealth was $ 42,523.

Panels A and B in Table 2 reports descriptive statistics for the consumption share and the portfolio shares of the assets of interest. Figure 2 displays these shares. These statistics and plots reveal several interesting stylized facts. First, the means of consumption and portfolio shares are all of the same order of magnitude and are strikingly small. Second, the consumption share is much smoother than the portfolio shares. Third, the shares for reserves and corporate equity exhibit the most pronounced fluctuations. Fourth, the consumption share displays strong comovements with all portfolio shares, except for corporate equity. Finally, the share for corporate equity is negatively correlated with noncorporate and home equity, whereas it is weakly correlated with the other assets. This last result is consistent with the cross-sectional findings that households more exposed to background risks – in the form of real estate and proprietary business – hold a lower share of their assets in stock (Heaton and Lucas 2000a, Heaton and Lucas 2000b). Also, these stylized facts suggest that the common practice of focusing mainly on corporate equity may not adequately characterize the actual portfolio. That is, this asset constitutes a very small proportion of total wealth, and its share displays atypically large volatility and small covolatility.

The empirical regularities also suggest a lesser importance for financial assets, and in particular for corporate equity — compared to that found in previous studies. For example, Ibbotson and Brinson (1987) deduce that the value of all corporate equity accounts for about 10.0% of total wealth, while Heaton and Lucas (1997) find that the average share for stocks held by representative (i.e. nonretired, medium-worth) investors is 9.4%. In contrast, our results highlight that the mean for the share of corporate equity is a meager 1.6%. In addition, Heaton and Lucas (1997) conclude that the representative agents' average shares are 8.0% for cash, 2.8% for pension, and 10.7% for proprietary business. Our findings indicate lower average shares for deposits, reserves, and noncorporate equity. Finally, Campbell (1996) assumes that the average share of financial assets is 33%, which roughly corresponds to the fraction of the gross national product that goes to capital. By comparison, our results reveal that the mean of the share for all financial assets is a modest 8.54% – given that the sum of the average shares for our selected financial assets is 7.17% and that the average aggregate value of our selected financial assets represents 84% of the total financial wealth.

Aside from Heaton and Lucas (1997) who extrapolate their findings from cross-sectional data, all the other estimates correspond more to informed guesses than actual measurements. In contrast, our approach is based on official aggregate data on asset stocks, and a realistic measure of total wealth.

Figure 3 plots the actual aggregate composition of risky assets. This composition is summarized by the relationship between the ratio of each asset to stock holdings and the empirical shares of corporate equity. Interestingly, this relationship is systematically downward-sloping, which is consistent with popular financial advisors' recommendations. It is typically recommended that high risk aversion (conservative) investors should keep a high share in cash, with the rest split up between stocks and bonds. For less risk averse (aggressive) investors, the recommended strategy is to reduce cash holdings, and increase stocks, while bonds are kept constant, or decrease. Canner et al. (1997) highlight these patterns and conjecture that these also hold in the aggregate given the popularity of these recommendations, but do not present evidence to support this claim.

1.3 Individual Returns

We now turn to the question of evaluating the rates of return associated with each asset of interest. Toward that aim, a natural method would be to use existing return indices, such as the S&P500 index as a measure of return on corporate equity. However, most of the considered assets have no known index series. For example, pension and insurance reserves are invested differently by fund managers. Defined benefit plans are invested at over 80% in equities and fixed-income securities, whereas defined contribution plans are invested mainly in guaranteed investment contracts (GIC's). In comparison, life insurance reserves are invested predominantly in bonds and mortgages (Fabozzi 1999). Obtaining a single return index for total pensions plus insurance would therefore prove impractical.

As an alternative, we use the implicit rates of return obtained through the following expression:

$$r_{i,t+1} = \frac{W_{i,t+1}}{\alpha_{i,t}(W_t - C_t)} - 1, \tag{5}$$

for $i = 1, \ldots, n$. Equation (5) can be interpreted in terms of the investor's budget constraint (1), by isolating $W_{i,t+1}$ and summing over all assets to yield $\sum_{i=1}^{N} W_{i,t+1} = [\sum_{i=1}^{N}(1 + r_{i,t+1})\alpha_{i,t}](W_t - C_t)$, or equivalently,

$W_{t+1} = (1 + r_{p,t+1})(W_t - C_t)$. A main advantage is that (5) provides a return series for each asset for which an aggregate stock measure $W_{i,t}$ is available, whether or not an index series exists. Moreover, for those assets for which indices do exist, a simple verification check is available, whereby we compare our implicit return to the index series.

The implicit individual returns obtained from (5) are calculated for $t = 2, \ldots, T$. To do so, we use the constructed shares for the relevant assets. We also use the real, per-capita, historical values of the specific assets, actual consumption, and constructed total wealth. For comparison purposes, the excess returns are evaluated for the portfolio as well as for each specific assets by subtracting the implicit returns from the riskfree rate.

Panels A and B of Table 3 presents summary statistics for the portfolio and individual excess returns, which are plotted in Figure 4. These statistics and plots reveal some interesting stylized facts regarding the first moment of excess returns. In particular, the means of excess returns are always positive. Also, the average portfolio excess return is systematically smaller than the average individual excess returns, except for noncorporate equity. Finally, the means of excess returns on deposits, noncorporate equity, and home equity are smaller than those on corporate equity and reserves.

Some important stylized facts also stand out for the second moments of excess returns. In general, the individual excess returns are slightly more volatile than the portfolio excess return. Also, the individual and portfolio excess returns covary almost always positively, as reflected by the positive total wealth portfolio betas of specific assets. These features do not hold, however, for corporate equity. The excess return for corporate equity is much more volatile than the portfolio excess return, and is, in fact, by far the most volatile of all the individual excess returns that we consider. Furthermore, the corporate equity excess return is the only one that covaries negatively with the portfolio excess return. It is worth noting that similar values for the beta on corporate equity were conjectured by other researchers (Ibbotson and Brinson 1987, Chou et al. 1992). Finally, among all the specific assets, the corporate-equity excess return exhibits the largest correlation (with the reserve excess return), the only negative correlation (with the noncorporate-equity excess return), and the smallest positive correlation (with the home-equity excess return). The finding that returns on corporate and home equities are almost completely uncorrelated has also been documented by other researchers (Goetzmann 1993, Flavin and Yamashita 1998).

These empirical regularities highlight the importance of Roll's critique on the pitfalls of using stock-market indices to proxy the portfolio return. Corporate-equity excess return displays atypical features on both first and second moments, so that it bears little relationship with the portfolio excess return. In particular, the portfolio excess return exhibits a strikingly more important degree of diversification.

These facts also illustrate the well-known difficulties in reconciling first and second moments of excess returns. For example, from a static CAPM perspective, the large mean excess return on corporate equity is not justified by its low beta. Moreover, observe that the excess returns on deposits and reserves are markedly different, although the betas are similar. Likewise, the excess returns on home equity is over 80% larger than on noncorporate assets, although the two betas are quite close. This suggests that some factors typically ignored in this static framework play an important role.

We showed earlier that our measured total wealth series and its corresponding return are realistic. A further diagnostic exercise is to simply compare the constructed excess return on corporate equity with the excess return on the S&P500 stock-market index. These two series are not totally comparable (only a subset of corporate firms are listed on the S&P500 index), but nonetheless, the constructed series tracks remarkably well the stock-index data (see Figure 5). Interestingly, the high degree of correspondence between our series and popular indices is not limited to stock. For example, Goetzmann (1993) finds that the average rate of return (standard deviation) to housing in Atlanta is 6.97% (3.48%), and 7.03% (4.85%) for Chicago between 1976 and 1986. The correlation with the S&P500 index is -18% for Atlanta, and -11% for Chicago. For the same sample period, our implicit rate of return on home equity is 7.04% (3.75%), while the correlation with the S&P500 is -17% . Overall, we strongly believe that this evidence and the earlier comparison results obtained from alternative measures of wealth put high confidence on the adequacy of our constructed series.

2 Theoretical Consumption and Portfolio Allocations

This section first presents the model and derives an analytical approximation of the optimal consumption and portfolio decision rules. We then explain our calibrations which we use to predict the shares.

2.1 Model

The model postulates that a representative investor chooses consumption and portfolio to maximize his utility subject to a budget constraint and a law of motion for the exogenous individual excess returns. More precisely, the investor is characterized by the preferences developed by Epstein and Zin (1989) and Weil (1990):

$$U_t = \left\{ (1-\delta) C_t^{\frac{\psi-1}{\psi}} + \delta \left(\mathrm{E}_t U_{t+1}^{1-\gamma} \right)^{\frac{\psi-1}{\psi(1-\gamma)}} \right\}^{\frac{\psi}{\psi-1}}, \tag{6}$$

where E_t represents the expectation operator conditional on information available in period t; U_t corresponds to the utility; and δ, γ, and ψ are the underlying preference parameters. The time discount factor $\delta \in (0, 1)$ captures the investor's impatience. The relative risk aversion $\gamma > 0$ summarizes the investor's attitudes towards atemporal risk. The elasticity of intertemporal substitution $\psi > 0$ translates the investor's attitudes towards intertemporal reallocation of consumption. Also, the restriction $\gamma = \psi^{-1}$ yields the standard state- and time-separable Von Neuman-Morgenstern preferences. In contrast, $\gamma \neq \psi^{-1}$ produces nonseparable preferences. The nonseparable case is of particular interest, since it allows the disentanglement of the role of the conceptually distinct attitudes towards risk and towards intertemporal substitution on the investor's optimal consumption and portfolio decisions.

The investor also faces the budget constraint (1). Recall that this constraint involves the wealth produced by all assets, including human capital. Finally, the investor takes into account the law of motion describing the dynamics of the exogenous individual excess returns. This law is specified as the following multivariate stochastic process:

$$\boldsymbol{r}_t^e = \boldsymbol{e} x_{t-1} + [(\boldsymbol{e}/\lambda)\eta_t + \boldsymbol{\nu}_t], \tag{7}$$

$$x_t = \mu(1-\phi) + \phi x_{t-1} + \eta_t, \tag{8}$$

$$\begin{pmatrix} \boldsymbol{r}_t^e - \mathrm{E}_{t-1}\boldsymbol{r}_t^e \\ x_t - \mathrm{E}_{t-1}x_t \end{pmatrix} \sim N.I.D. \left[\begin{pmatrix} \boldsymbol{0} \\ 0 \end{pmatrix}, \begin{pmatrix} \boldsymbol{\sigma} & \boldsymbol{\sigma}_\eta \\ \boldsymbol{\sigma}_\eta & \sigma_\eta^2 \end{pmatrix} \right], \tag{9}$$

where $\boldsymbol{r}_t^e$ is the n-vector of individual excess returns, $\boldsymbol{\nu}_t$ is the n-vector of zero-mean idiosyncratic terms, x_t is a common factor, and η_t is its zero-mean innovation. The underlying parameters are λ, μ, ϕ, and σ_η^2, as well as the additional elements included in the vectors $\boldsymbol{e}$ and $\boldsymbol{\sigma}_\eta$ and in the matrix $\boldsymbol{\sigma}$. Here, $\boldsymbol{\sigma}_\eta = (\boldsymbol{e}/\lambda)\sigma_\eta^2$ captures the conditional covariances between excess returns and the common factor. Also, $\boldsymbol{\sigma} = (\boldsymbol{e}/\lambda)\sigma_\eta^2(\boldsymbol{e}/\lambda)' + \boldsymbol{\omega}$ corresponds to the nondiagonal conditional covariance matrix of excess returns and $\boldsymbol{\omega}$ is the diagonal conditional covariance matrix of the idiosyncratic terms.

Note that equation (7) decomposes excess returns in terms of an anticipated component and an unanticipated portion (i.e. the expression between brackets). The anticipated term is determined by lagged values of the factor. The unanticipated component is related to the current innovations in the factor and the contemporaneous idiosyncratic terms. Also, equation (8) stipulates that the factor is governed by a univariate first-order autoregressive process. In the spirit of Merton (1971, 1973), this factor corresponds to the underlying state of the world, and as such, captures the systematic risk. Hence, the loadings $\boldsymbol{e} = (\boldsymbol{e}/\lambda) \times \lambda$ summarize both the characteristics $(\boldsymbol{e}/\lambda)$ and the price λ of the systematic risk. Finally, the expression (9) simplifies the investor's problem by assuming that innovations in excess returns and in the state follow a joint conditional homoscedastic Gaussian distribution. Recall that this scedastic structure is likely to hold given that our individual excess returns are recorded at an annual frequency.

Unfortunately, the investor's problem just described has no exact analytical solution for general values of the underlying parameters. We circumvent this problem by using the analytical approximation derived in Appendix A. In brief, this approximation first relates the portfolio return to individual returns, where this relationship holds exactly in continuous time. The approximation also log-linearizes the budget constraint around the unconditional mean of the log consumption share. Campbell (1993) and Campbell and Koo (1997) show that this linearization is exact when the consumption share is constant (i.e. when $\boldsymbol{e} = 0$, $\phi = 0$, or $\psi = 1$), and is accurate as long as this share remains smooth. Finally, the approximation requires a second-order Taylor expansion to the Euler equations around the conditional means of consumption growth and returns. Campbell and Viceira (1999) point out that this expansion is exact when these variables are conditionally lognormally distributed (i.e. when $\psi = 1$).

A convenient property of the approximation used here is that the optimal log consumption and portfolio shares are quadratic and linear respectively in the state:

$$\log \alpha_{c,t} = b_0 + b_1 x_t + b_2 x_t^2, \tag{10}$$

$$\boldsymbol{\alpha}_t = \boldsymbol{a}_0 + \boldsymbol{a}_1 x_t. \tag{11}$$

Here, b_0, b_1, and b_2 are obtained by solving a recursive nonlinear equation system whose coefficients are complex functions of the underlying parameters. The elements in $\boldsymbol{a}_0$ and $\boldsymbol{a}_1$ are also function of these parameters and of b_0, b_1, and b_2.

Interestingly, Campbell, Cocco, Gomes, Maenhout and Viceira (1998) demonstrate that the single risky asset version of the approximate consumption and portfolio optimal decision rules (10) and (11) are very accurate, except for extremely large positive values of the state. Furthermore, evaluating the decision rules (10) and (11) at $\boldsymbol{e} = 0$ and/or $\phi = 0$ leads to the known exact analytical solution obtained from constant investment opportunity sets; that is, the investor faces excess returns that are conditionally independently and identically distributed. Also, restricting (10) to $\psi = 1$ produces the exact myopic consumption rule, that is, the investor consumes each period the same fraction of its wealth. Evaluating (11) at $\gamma = 1$ yields the exact myopic portfolio rules; that is, the investor never takes hedging positions (Giovannini and Weil 1989). Note that this last behavior occurs because the parameters of the decision rules are not affected by $\boldsymbol{\sigma}_\eta$ (Campbell and Viceira 1999).

2.2 Calibration

In our calibrations, we set values for the preference parameters. As is standard practice, we choose the time discount factor $\delta = 0.96$, so that the implied annual (net) time discount rate is 4.0%. We also select the various values $\gamma = (1, 4, 10, 40)$ for the relative risk aversion and $\psi = (1, 1/4, 1/10, 1/40)$ for the elasticity of intertemporal substitution. This permits us to study the behavior of the shares induced by separable preferences ($\gamma = \psi^{-1}$) and nonseparable preferences ($\gamma \neq \psi^{-1}$). Moreover, this allows us to perform a sensitivity analysis to understand how the key preference parameters affect the optimal consumption and portfolio shares. This analysis is required to document the optimal allocations predicted by myopic (nonmyopic) consumption rules, i.e. $\psi = 1$ ($\psi \neq 1$), and myopic (nonmyopic) portfolio rules, i.e. $\gamma = 1$ ($\gamma \neq 1$). This analysis is also useful to describe the behavior of an investor that is either moderately, highly, or extremely risk averse, i.e. $\gamma = 1$, $\gamma > 1$, or $\gamma = 40$.

We next select values for the parameter $\rho \equiv 1 - \exp[\mathrm{E}\log(\alpha_{c,t})]$ of the log-linearized budget constraint. For the myopic consumption rule, we use the exact analytical result stating that $\rho = \delta$. For the nonmyopic consumption rule, ρ becomes a function of the endogenous consumption choice and is determined from the recursive procedure outlined in Campbell and Viceira (1999).

We also set values for the average of the riskfree rate r_f as well as for the parameters of the excess-return stochastic process (7–9). We calibrate $r_f = 0.77\%$ from the sample mean of our annual real riskfree rate series. In addition, we fix the stochastic-process parameters to the estimates that are obtained from the maximum-likelihood procedure detailed in Appendix B. This procedure also produces a Kalman-filter estimate of the state, which is treated as a latent variable. It is worth noting that our latent-variable approach departs from the standard practice of assuming that the state exactly corresponds to some observable series. Importantly, our approach is motivated by the lack of empirical evidence on the existence of published series which have a joint predictive power for all the individual excess returns that we consider. For example, Campbell et al. (1999) find that the dividend-price ratio used in numerous studies as a proxy of the state only Granger-causes the stock-index excess return.

Table 4 presents the estimates of the stochastic-process parameters. Note that the estimates of the loadings $\boldsymbol{e}$ and of the price of risk λ are all statistically positive. Moreover, the estimates of the standard errors $\boldsymbol{\omega}^{1/2}$ are always significant, and suggest that the idiosyncratic risk is quantitatively much larger for corporate equity. Finally, the estimate of the mean μ of the state variable is positive, although not significant; while the estimate of the first-order autocorrelation ϕ is positive and significant.

Importantly, these estimates imply that the set of investment opportunities is not constant, since expected individual excess returns are statistically time-varying. As already mentioned, this feature rules out an important class of parameterization for which exact analytical solutions are known. Moreover, the estimates reveal that investment opportunities improve when the latent variable increases, so that high values of x_t tend to be associated with favorable states of the world. More precisely, improvements occur because individual excess returns are contemporaneously high when they are expected to rise in the future. To see this, first note that current and future values of the state increase following a positive shock in its innovation. This leads to positive revisions in current individual excess returns, since they conditionally covary positively with the innovations in the state — as implied by our estimates. This also leads to expected increases in future excess returns, since they are positively related to lagged values of the state.

Figure 4 displays the individual excess returns, as already mentioned, as well as our estimate of the state. These plots confirm that these returns are adequately characterized by our latent variable. In particular, the turning points of excess returns often precisely coincide with those of the state. Also, the fluctuations in excess returns are about the same order of magnitude than those in the latent variable. One exception, however, is the greater fluctuations in the corporate-equity excess return, which simply reflects the stylized fact that this return is the most volatile.

Figure 6 confronts our latent variable to the actual dividend yield on the S&P500 stock-market index, a proxy of the state commonly used. A striking feature is that our estimate of the state roughly depicts the mirror image of the S&P500 dividend-price ratio. This suggests that this ratio is positively autocorrelated, as for our estimate of the state, and is conditionally negatively correlated with individual excess returns, as opposed to our latent variable. The additional well-documented evidence that the lagged dividend-price series positively affects current stock-index excess returns implies that these returns are contemporaneously high when they are expected to fall in the future. This sharply contrasts with the behavior obtained from our estimate of the state. In this sense, the dividend-price ratio and our latent variable exhibit different time-series properties.

2.3 Predicted Shares

Table 5 reports the means of predicted consumption and portfolio shares for the calibrations just presented. Table 6 shows the predicted coefficients of the optimal decision rules. These tables reveal that, in general, the means of consumption shares predicted by separable preferences considerably differ with those induced by nonseparable preferences. This occurs because the consumption share is sensitive to both relative risk aversion and the elasticity of intertemporal substitution. For example, the average consumption share is usually decreasing in relative risk aversion. It is also decreasing in the elasticity of intertemporal substitution, except for extremely high levels of relative risk aversions. In concrete terms, the myopic consumption rule predicts that the average shares are equal to the time discount rate of 4% — regardless of the value for relative risk aversion. For the nonmyopic consumption rule, the average shares substantially increase (decrease) to reach limiting values of 81% (2%) when the investor is moderately (extremely) risk averse.

This complex pattern can be understood from the predicted coefficients of the consumption rule. First, the myopic consumption rule always implies that b_1 and b_2 are zero. This reflects the well-known exact cancellation of the intertemporal substitution and income effects associated with changes in investment opportunities. Second, the nonmyopic consumption rule for a moderately risk-averse investor results in b_1 and b_2 being positive. This indicates that the investor's current consumption systematically increases when investment opportunities improve; that is, the income effect dominates no matter which state of the world prevails. Third, the nonmyopic consumption rule for a highly risk-averse investor reveals that b_1 is negative and b_2 is positive. This implies that the consumption share decreases (increases) following investment opportunity improvements that occur when x_t is small (large); that is, the intertemporal substitution (income) effect dominates in unfavorable (favorable) states of the world. Given our estimates of x_t however, the income effect dominates most of the time, so that consumption is positively correlated with the state variable. Finally, the absolute values of b_1 and b_2 tend to decrease in relative risk aversion. This occurs because a highly risk-averse investor prefers to reduce his consumption exposure to changes in the states of the world.

At this point, it is worth noting that our predicted consumption shares display qualitatively similar patterns to those reported in previous studies. In fact, the quantitative differences for the shares associated with the myopic consumption rule are entirely due to alternative assumed values for the time discount rate. The numerical differences for the shares induced by the nonmyopic consumption rule are also explained by alternative specifications of the excess-return process. More precisely, Campbell and Viceira (1999) relate the single risky excess return on corporate equity to the dividend-price ratio, where this state variable is assumed to follow a univariate first-order autoregressive process. Also, Campbell et al. (1999) specify a first-order vector autoregressive process involving returns on cash, stock, and bond, as well as the (relative) bill rate, the yield spread, and the dividend-price ratio to show that this last series represents the state variable that impacts the most on the investor's behavior. As already discussed, however, the time-series properties of the dividend-price ratio differ from those describing our latent variable.

We next focus on the means of the corporate equity shares. Note that the means predicted by separable and nonseparable preferences are very similar. This arises because these means are greatly affected by relative risk aversion and are almost completely unaltered by the elasticity of intertemporal substitution. In particular, the corporate-equity share decreases rapidly in relative risk aversion. For example, a moderately risk-averse agent invests slightly more than the totality of his wealth in corporate equities, while an extremely risk-averse investor allocates only 2.3% of his wealth in these stocks.

This demand behavior can be explained from the predicted coefficients of the portfolio rules. First, note that the coefficients $a_{0,inc}$ and $a_{1,inc}$ are extremely sensitive to relative risk aversion and are insensitive to the elasticity of intertemporal substitution. This confirms the notion that relative risk aversion constitutes a prime determinant of the demand for corporate equity. Second, the myopic portfolio rule reveals that $a_{0,inc}$ and $a_{1,inc}$ are positive. It follows that the myopic demand for corporate equity is positive on average (since the mean of the state variable exceeds zero) and that it is smaller (larger) in unfavorable (favorable) states of the world. Third, nonmyopic portfolio rule leads to substantially smaller positive values for $a_{0,inc}$ and $a_{1,inc}$. This indicates that the total demand for corporate equity is smaller than its myopic demand. This implies that the hedging demand is negative for any given state of the world, or synonymously, that the investor takes a short position on the hedging component of his total portfolio. As a result, a highly risk-averse investor prefers to reduce his exposure in this risky asset, in all states of the world.

Our central finding that the demand for corporate equity is substantially driven by relative risk aversion concurs with those of earlier studies. However, our predicted means for the share of these stocks decline much more rapidly in relative risk aversion. For example, we show that the average shares are 5.1 and 51.3 times smaller for $\gamma = 4$ and $\gamma = 40$ than for $\gamma = 1$, when the state is defined from our latent-variable approach. In contrast, Campbell and Viceira (1999) find that these ratios are only 1.3 and 7.5, when the state is proxied by the dividend-price ratio. Furthermore, Campbell et al. (1999) establish that the inclusion of this ratio as a state almost triples the stock total demand and contributes for 90% of variation in its hedging demand. These features can be rationalized by first recalling that the dividend-price ratio implies that a rise in current excess return signals declines in future returns, while it indicates an increase in expected future returns, according to our latent-variable approach. Hence, the dividend-price ratio leads to a larger hedging demand than that induced by our estimate of the state. Moreover, the difference between these hedging demands increases in relative risk aversion, since a highly risk-averse investor prefers to hold assets that deliver wealth when investment opportunities deteriorate.

We finally present the means of our other asset shares. Again, the means induced by separable and nonseparable preferences are identical, which indicates that they are primarily determined by relative risk aversion. Also, these shares decline rapidly as relative risk aversion increases. In concrete terms, a moderately risk-averse investor places between 881% and 1893% of his wealth in our selected assets, while an extremely risk-averse investor attributes between 14% and 29% of his wealth in these assets.

The predicted coefficients $a_{0,i}$ and $a_{1,i}$ of the portfolio rules are always closely related to relative risk aversion, which confirms that the investor's portfolio decisions are crucially affected by his attitude towards risk. Also, the predicted coefficients imply that all our selected asset total demands decrease (increase) in unfavorable (favorable) states of the world, while hedging demands are negative in all states of the world. Finally, these coefficients indicate that a highly risk-averse agent invests less in each risky asset, for any given state of the world.

Unfortunately, these results cannot be compared with previous work. More precisely, Campbell and Viceira (1999) limit their portfolio analysis to stocks, while Campbell et al. (1999) incorporate additional risky assets that are not the ones that we consider. Yet, it is instructive to confront our findings obtained for the various assets. This exercise reveals that the average shares are always positive, so that the investor systematically takes long positions in the total demand for all our selected assets. This is due to the fact that excess returns tend to move in the same direction, as suggested by the positive covariances obtained for all pairs of shocks to excess returns (see Table 4). However, the average share for other assets are much larger than those for corporate equity, especially when risk aversion is small. Hence, the ratio of other assets to corporate equity decreases in risk aversion. This implies a positive relationship between this ratio and the share for corporate equity, for fixed values of elasticity of intertemporal substitution and state (see Figure 7).

More precisely, a moderately risk-averse agent has a smaller myopic demand for corporate equity because its Sharpe ratio (SR_t) is much smaller (see Table 4). Moreover, a highly risk-averse investor reduces his demand for other assets considerably more than for corporate equity. This occurs because the other assets are worse hedges against deteriorations in the investment opportunities. Following Merton (1973), the weight of the hedging portfolio relative to the myopic portfolio in total asset demand increases in risk aversion. Furthermore, recall that consumption covaries positively with the state, while the returns for other assets (and to a lesser extent for corporate equity) covary positively with the state (see Table 4). Consequently, the highly risk-averse investor takes a short position on his hedging portfolio for the other assets (and to a lesser extent for corporate equity); the other assets are less attractive since they produce much lower returns when the investment opportunities deteriorate and consumption is also low.

To summarize, a moderately risk-averse investor substantially increases consumption as well as portfolio shares in our selected risky assets, and reduces shares in other less risky assets, when investment opportunity improvements either occur in favorable or unfavorable states of the world. Thus, a moderately risk-averse investor tends to hedge his long positions in more risky assets by taking short positions in more certain assets. By comparison, a highly risk-averse investor also increases (although to a lesser extent) consumption and risky-asset shares, following investment opportunity improvements that arise in favorable states. In contrast, a highly risk-averse investor usually prefers to reduce his consumption in favor of saving, when investment opportunity improvements occur in unfavorable states.

3 Comparison of Empirical and Theoretical Allocations

In this section, we formally test whether the predicted optimal decision rules accord with stylized facts. We subsequently discuss the economic interpretation of the findings and propose some extensions.

3.1 Results

Table 5 reports the average predicted shares, the average empirical shares, and the levels of significance that these shares are identical. Similarly, Table 6 presents the predicted coefficients of the optimal decision rules, the ordinary least squares estimated coefficients of the unrestricted reduced forms for (10)–(11), and the levels of significance that these coefficients are the same. These tables denote the 15%, 10%, and 5% levels of significance by one, two, and three asterisks, respectively. All these levels are associated with $\chi^2(1)$ distributed Wald test statistics which take into account the uncertainty related to the estimates of the multivariate stochastic excess-return process (7)–(9).

First, note that in general the predicted means of the consumption share numerically and statistically over-predict their empirical counterparts. This finding holds for almost all calibrations of the preference parameters. Exceptions are obtained when $\gamma = 40$ is combined with $\psi = 1/10$ or $1/40$, such that the investor is extremely risk averse and very reluctant to substitute consumption intertemporally.

The predicted coefficients of the consumption decision rule are also often statistically different from the corresponding estimates of the unrestricted reduced forms. More specifically, the predictions for b_0 are most of the time significantly larger, the predictions for b_1 are always statistically larger and even display occasionally the wrong sign, while those for b_2 are sometimes statistically different. Consequently, the theoretical consumption allocations never capture adequately the empirical intertemporal substitution and income effects associated with changes in investment opportunities.

Second, the means of the predicted corporate-equity share often numerically and statistically exceed those found in the data. This result is verified for most values of the preference parameters. Exceptions emerge as long as $\gamma = 40$, regardless of the values of ψ; that is, the investor remains extremely risk averse, but can either be inclined or reluctant to substitute consumption intertemporally.

The predicted coefficients of the portfolio decision rule for corporate equity are always significantly different from the associated estimates of the unrestricted reduced forms. In particular, the predictions for $a_{0,inc}$ are statistically larger for a moderately or highly risk-averse investor and smaller for an extremely risk-averse agent, while the predictions for $a_{1,inc}$ are always significantly larger. As a result, the predicted portfolio allocations never disentangle appropriately the empirical myopic and hedging demands for corporate equity.

Third, the means of the predicted portfolio shares for the other assets grossly over-estimate the empirical shares. As for corporate equity, the predicted shares for deposits and reserves are insignificantly different from actual means only when $\gamma = 40$ – so once again the investor is extremely risk averse. In contrast, the predicted shares for noncorporate and home equities are always significantly different, even when the investor is extremely risk averse. In this sense, the model performance for noncorporate and home equities is poorer than that obtained for our alternative assets, and in particular for corporate equity.

Fourth, the predicted coefficients of the portfolio rules for deposits, reserves, as well as noncorporate and home equities are often statistically different from their empirical counterparts. The predicted $a_{0,i}$ are most of the time significantly larger (smaller) at low (high) relative risk aversion, whereas the predicted $a_{1,i}$ are always numerically larger and have occasionally the wrong sign. Hence, the predicted portfolio allocations for these assets do not characterize accurately the empirical myopic and hedging demands, as was the case for corporate equity.

Finally, contrasting Figure 7 with Figure 3 reveals that the predicted portfolio mix does not accord with the actual aggregate mix. More precisely, the predicted ratio of each asset relative to stock holdings is increasing in corporate equity share, whereas the opposite relationship is observed in the data.

3.2 Discussion

The findings just presented reveal that no combination of the key preference parameters allow the joint replications of the empirical consumption and portfolio allocations; all calibrations of the exercise lead to important anomalies regarding the decomposition of total asset demands into myopic and hedging behaviors as well as the disentanglement of intertemporal substitution and income effects associated with changes in investment opportunities. Also, most parameterizations produce means of consumption and portfolio shares that are inconsistent with observed average shares. In fact, the calibrations combining $\psi = 1/10$ or $\psi = 1/40$ with $\gamma = 40$ are the only one that provide adequate descriptions of the actual average shares for consumption and for some assets — namely deposits, reserves, and corporate equity. In this respect, these results are the most interesting.

These parameter combinations provide support for the hypothesis that $\gamma = \psi^{-1}$, so that the validity of separable preferences cannot be refuted. This is consistent with the empirical asset returns' literature, such as the simulation results in Weil (1989) and the test results in Jorion and Giovannini (1993) and Normandin and St-Amour (1998). Consequently, incorporating nonseparable preferences adds little to our understanding of empirical consumption and portfolio shares. Furthermore, separable preferences imply that fluctuations in the intertemporal marginal rates of substitution can only be ascribed to movements in consumption. It follows that expected excess returns are solely determined by the consumption risk; the market beta plays no role because it is not priced.

The most interesting calibrations involve extremely low values of ψ, i.e. the investor is highly reluctant to substitute consumption across periods. Interestingly, this concurs with the fact that historical aggregate consumption growth is smooth (Deaton 1987, Campbell and Deaton 1989). Moreover, this feature reflects the condition required to reconcile changes in actual aggregate consumption to the pronounced fluctuations in historical real interest rate (Hall 1988, Campbell and Mankiw 1989). Finally, this finding can be related to the riskfree rate puzzle (Weil 1989). This puzzle highlights the difficulty to rationalize the low mean of actual real short-term interest rate, given high mean growth rate of consumption.

The best calibrations also involve large values of γ, i.e. the investor is extremely risk averse. This feature is the counterpart of the equity premium puzzle (Mehra and Prescott 1985). This puzzle states that the high average stock-index excess returns can be explained by the measured quantity of consumption risk, only when the assumed values of relative risk aversion are unreasonably high. This result also parallels the stockholding puzzle (Mankiw and Zeldes 1991, Haliassos and Bertaut 1995). This puzzle highlights the notion that the small observed proportion of the population holding stocks directly can be reconciled with the historical corporate-equity excess return only through implausible levels of risk aversion. Importantly, our results suggest that our analogue of the stockholding puzzle is not unique to corporate equity; it rather extends to all other assets: the low actual portfolio shares for these assets are always significantly over-predicted using low values of relative risk aversion. This anomaly is in fact more severe for noncorporate and home equities, given that the associated empirical shares are statistically over-evaluated even under extremely high relative risk aversion.

Finally, calibrations with different levels of risk aversion lead to the asset allocation puzzle: the model is unable to reproduce the observed portfolio mix. Our results do not support the conjecture by Canner et al. (1997) that using richer specifications for preferences and investment opportunities potentially allows to replicate the observed pattern of portfolio decisions. Our findings are also different from Brennan and Xia

(2000) because our model allows for dynamic hedging on *all* securities, rather than only on a subset of assets. Note however that our results are in line with those of Elton and Gruber (2000), who show that allowing short sales may produce a monotonically decreasing or increasing relationship for the portfolio mix.

Overall, further research will need to address the additional anomalies uncovered in our results. In particular, attention should be devoted to the behavior of the demand for assets *other* than corporate equity. Efforts should also concentrate on the fundamental forces of intertemporal substitution and hedging that underlie consumption and portfolio allocations.

A Analytical Approximation of Optimal Decision Rules

This appendix details the complete derivation for the analytical approximation of the optimal consumption and portfolio rules. This derivation extend the approach outlined in Campbell and Viceira (1999) to a multi-risky-asset environment.

A.1 Intermediate results

Lemma 1 *The state variable is characterized by:*

$$\begin{aligned}
\mathrm{E}_{t+1}x_{t+j} &= \mu + \phi^j(x_t-\mu) + \phi^{j-1}\eta_{t+1}, \\
\mathrm{E}_{t+1}x_{t+j}^2 &= \mu^2(1-\phi^j)^2 + \tfrac{1-\phi^{2(j-1)}}{1-\phi^2}\sigma_\eta^2 + 2\mu\phi^j(1-\phi^j)x_t + \phi^{2j}x_t^2 \\
&\quad + \phi^{2(j-1)}\eta_{t+1}^2 + 2\phi^{1+2(j-1)}(x_t-\mu)\eta_{t+1} + 2\mu\phi^{j-1}\eta_{t+1}, \\
x_{t+1}^2 - \mathrm{E}_t x_{t+1}^2 &= (\eta_{t+1}^2 - \sigma_\eta^2) + 2[\mu(1-\phi)+\phi x_t]\eta_{t+1}.
\end{aligned}$$

Proof. See Campbell and Viceira (1999), proof to Lemmas 1 and 2. ∎

Lemma 2 *Denote by $dV(t)/V(t)$ the instantaneous (net) return on the portfolio, with (net) individual returns denoted $dP_i(t)/P_i(t)$, such that,*

$$\frac{dV(t)}{V(t)} = \sum_i \alpha_i(t)\frac{dP_i(t)}{P_i(t)},$$

where individual returns follow an Itô process:

$$\frac{dP_i(t)}{P_i(t)} = \mu_i(t)dt + \sigma_i(t)dZ_i(t),$$

and where $dZ_i(t)$ is a correlated Wiener process satisfying $\sigma_{ij}(t) = \sigma_i(t)dZ_i(t)\sigma_j(t)dZ_j(t)$. Let $v(t) \equiv \log V(t)$, then $dv(t)$, is characterized by:

$$dv(t) = \sum_i \alpha_i(t)[dp_i(t) + 0.5\sigma_{ii}(t)] - 0.5\sum_i\sum_j \alpha_i(t)\alpha_j(t)\sigma_{ij}(t). \tag{12}$$

Proof. By Itô's lemma, we find that:

$$\begin{aligned}
dv(t) &\equiv d\log[V(t)], \\
&= \tfrac{dV}{V} - 0.5\left(\tfrac{dV}{V}\right)^2, \\
&= \textstyle\sum_i \alpha_i(t)\frac{dP_i(t)}{P_i(t)} - 0.5\sum_i\sum_j \alpha_i(t)\alpha_j(t)\left[\frac{dP_i(t)}{P_i(t)}\frac{dP_j(t)}{P_j(t)}\right], \\
&= \textstyle\sum_i \alpha_i(t)\frac{dP_i(t)}{P_i(t)} - 0.5\sum_i\sum_j \alpha_i(t)\alpha_j(t)\sigma_{ij}(t).
\end{aligned} \tag{13}$$

Similarly, by Itô's lemma, we have that $dp_i(t) \equiv d\log[P_i(t)] = dP_i(t)/P_i(t) - 0.5[dP_i(t)/P_i(t)]^2$, such that,

$$\frac{dP_i(t)}{P_i(t)} = dp_i(t) + 0.5\sigma_{ii}(t).$$

Substitute this last equation in (13) to obtain (12). ∎

Lemma 3 *Conditional expected portfolio returns and innovations are given as:*

$$\begin{aligned}
\mathrm{E}_t r_{p,t+1} &= r_f + p_0 + p_1 x_t + p_2 x_t^2, \\
r_{p,t+1} - \mathrm{E}_t r_{p,t+1} &= \textstyle\sum_i (a_{0i} + a_{1i}x_t)u_{i,t+1},
\end{aligned}$$

where,

$$\begin{aligned}
p_0 &\equiv 0.5\textstyle\sum_i a_{0i}\sigma_{ii} - 0.5\sum_i\sum_j a_{0i}a_{0j}\sigma_{ij}, \\
p_1 &\equiv 0.5\textstyle\sum_i a_{1i}\sigma_{ii} + \sum_i a_{0i}e_i - 0.5\sum_i\sum_j(a_{0j}a_{1i} + a_{0i}a_{1j})\sigma_{ij}, \\
p_2 &\equiv \textstyle\sum_i a_{1i}e_i - 0.5\sum_i\sum_j a_{1i}a_{1j}\sigma_{ij}.
\end{aligned}$$

and,

$$u_{i,t+1} = (e_i/\lambda)\eta_{t+1} + \nu_{i,t+1}.$$

Proof. From the process (7), candidate solution (11), and portfolio returns (12),

$$\begin{aligned}
\mathrm{E}_t r_{p,t+1} &= \textstyle\sum_i \alpha_{i,t}\left[\mathrm{E}_t r_{i,t+1} + 0.5\sigma_{ii}\right] - 0.5\sum_i\sum_j \alpha_{i,t}\alpha_{j,t}\sigma_{ij}, \\
&= r_f + \textstyle\sum_i (a_{0i} + a_{1i}x_t)(e_i x_t + 0.5\sigma_{ii}) - 0.5\sum_i\sum_j (a_{0i} + a_{1i}x_t)(a_{0j} + a_{1j}x_t)\sigma_{ij}, \\
\mathrm{E}_t r_{p,t+1} &= r_f + p_0 + p_1 x_t + p_2 x_t^2,
\end{aligned}$$

with p_0, p_1, and p_2 given as in Lemma 3. Observe that under conditional homoscedasticity, the p's are time-invariant. Moreover,

$$\begin{aligned}
\mathrm{E}_t r_{p,t+1} - r_{p,t+1} &= \textstyle\sum_i \alpha_{i,t}(r_{i,t+1} - \mathrm{E}_t r_{i,t+1}), \\
&= \textstyle\sum_i (a_{0i} + a_{1i}x_t)u_{i,t+1}.
\end{aligned}$$

∎

Lemma 4 *Consumption growth is characterized by:*

$$\mathrm{E}_t \Delta c_{t+1} = c_0 + c_1 x_t + c_2 x_t^2,$$

where,

$$\begin{aligned}
c_0 &\equiv b_0\left(\tfrac{\rho-1}{\rho}\right) + b_1\mu(1-\phi) + b_2\mu^2(1-\phi)^2 + b_2\sigma_\eta^2 + k + r_f + p_0, \\
c_1 &\equiv b_1\left(\tfrac{\rho\phi-1}{\rho}\right) + 2b_2\mu\phi(1-\phi) + p_1, \\
c_2 &\equiv b_2\left(\tfrac{\rho\phi^2-1}{\rho}\right) + p_2,
\end{aligned}$$

Δc_{t+1} *is consumption growth,* $k \equiv \log(\rho) + \frac{(1-\rho)}{\rho}\log(1-\rho)$ *and* $\rho \equiv 1 - \exp\{\mathrm{E}(\log\alpha_{c,t})\}$. *Moreover,*

$$\begin{aligned}
\Delta c_{t+1} - \mathrm{E}_t\Delta c_{t+1} &= [b_1 + 2b_2\mu(1-\phi)]\eta_{t+1} + b^2(\eta_{t+1}^2 - \sigma_\eta^2) + \textstyle\sum_i a_{0i}u_{i,t+1} \\
&\quad + x_t\left(2b_2\phi\eta_{t+1} + \textstyle\sum_i a_{1i}u_{i,t+1}\right).
\end{aligned}$$

Proof. From log-linearizing the budget constraint around the unconditional mean of log consumption share (Campbell and Viceira 1999, eqs. 15, 17):

$$\Delta c_{t+1} = \log\alpha_{c,t+1} - \rho^{-1}\log\alpha_{c,t} + k + r_{p,t+1}.$$

Use candidate solution (10), Lemma 1, and mean portfolio returns in Lemma 3 to obtain that

$$\begin{aligned}
\mathrm{E}_t\Delta c_{t+1} &= [b_0 - (1/\rho)b_0] + [b_1\mathrm{E}_t x_{t+1} - (1/\rho)b_1 x_t] + [b_2\mathrm{E}_t x_{t+1}^2 - (1/\rho)b_2 x_t^2] \\
&\quad + k + r_f + p_0 + p_1 x_t + p_2 x_t^2, \\
&= \left(\tfrac{\rho-1}{\rho}\right)b_0 + b_1\mu + b_1\phi(x_t-\mu) - (b_1/\rho)x_t \\
&\quad + b_2[\mu^2 + 2\mu\phi(x_t-\mu) + \phi^2(x_t-\mu)^2 + \sigma_\eta^2] - (1/\rho)b_2x_t^2 \\
&\quad + k + r_f + p_0 + p_1 x_t + p_2 x_t^2, \\
&= c_0 + c_1 x_t + c_2 x_t^2,
\end{aligned}$$

where c_0, c_1, c_2 are as defined.

Moreover, use candidate solution (10) and Lemma 3 to obtain that:

$$\begin{aligned}
\Delta c_{t+1} - \mathrm{E}_t\Delta c_{t+1} &= [b_1 + 2b_2\mu + 2b_2\phi(x_t-\mu)]\eta_{t+1} + b_2\eta_{t+1}^2 - b_2\sigma_\eta^2 \\
&\quad + \textstyle\sum_i (a_{0i} + a_{1i}x_t)u_{i,t+1},
\end{aligned}$$

which simplifies as stated. ∎

Lemma 5 *Let* $\tau_{p,t} \equiv \mathrm{Var}_t(\Delta c_{t+1} - \psi r_{p,t+1})$. *Then,* $\tau_{p,t}$ *is quadratic in the state variable:*

$$\tau_{p,t} = \tau_0 + \tau_1 x_t + \tau_2 x_t^2,$$

where,

$$\begin{aligned}
\tau_0 &\equiv -0.5(1-\gamma)/(1-\psi)\{[b_1^2+4b_1b_2\mu(1-\phi)+4b_2^2\mu_2^2(1-\phi)^2]\sigma_\eta^2-b_2^2(\sigma_\eta^2)^2 \\
&\quad +2(1-\psi)\textstyle\sum_i[b_1a_{0i}+2b_2\mu(1-\phi)a_{0i}]\sigma_{\eta i}+(1-\psi)^2\sum_i\sum_j a_{0i}a_{0j}\sigma_{ij}\}, \\
\tau_1 &\equiv -0.5(1-\gamma)/(1-\psi)\{[4b_1b_2\phi+8b_2^2\phi\mu(1-\phi)]\sigma_\eta^2 \\
&\quad +2(1-\psi)\textstyle\sum_i[b_1a_{1i}+2b_2\phi a_{0i}+2b_2\mu(1-\phi)a_{1i}]\sigma_{\eta i}+(1-\psi)^2\sum_i\sum_j(a_{0i}a_{1j}+a_{0j}a_{1i})\sigma_{ij}\}, \\
\tau_2 &\equiv -0.5(1-\gamma)/(1-\psi)\{4b_2^2\phi^2\sigma_\eta^2+4(1-\psi)\textstyle\sum_i b_2\phi a_{1i}\sigma_{\eta i}+(1-\psi)^2\sum_i\sum_j a_{1i}a_{1j}\sigma_{ij}\}.
\end{aligned}$$

Proof. Using Lemmas 3 and 4, and the Gaussian assumption, we can write $\tau_{p,t}$ as:

$$\begin{aligned}
\tau_{p,t} &= -0.5(1-\gamma)/(1-\psi)\mathrm{E}_t\{[(\Delta c_{t+1}-\mathrm{E}_t\Delta c_{t+1})-\psi(r_{p,t+1}-\mathrm{E}_t r_{p,t+1})]^2\}, \\
&= -0.5(1-\gamma)/(1-\psi)\mathrm{E}_t\{[b_1\eta_{t+1}+b_2(2[\mu+\phi(x_t-\mu)]\eta_{t+1}+\eta_{t+1}^2-\sigma_\eta^2) \\
&\quad +(1-\psi)\textstyle\sum_i(a_{0i}+a_{1i}x_t)u_{i,t+1}]^2\}, \\
&= -0.5(1-\gamma)/(1-\psi)\{(b_1+2b_2[\phi x_t+\mu(1-\phi)])^2\sigma_\eta^2-b_2^2(\sigma_\eta^2)^2 \\
&\quad +2(1-\psi)\textstyle\sum_i(b_1+2b_2[\phi x_t+\mu(1-\phi)])(a_{0i}+a_{1i}x_t)\sigma_{\eta i} \\
&\quad +(1-\psi)^2\textstyle\sum_i\sum_j(a_{0i}+a_{1i}x_t)(a_{0j}+a_{1j}x_t)\sigma_{ij}\}, \\
&= \tau_0+\tau_1x_t+\tau_2x_t^2,
\end{aligned}$$

where τ_0, τ_1, τ_2 are as stated. ∎

Lemma 6 *The parameters τ_0, τ_1, τ_2 in Lemma 5 also satisfy:*

$$\begin{aligned}
\tau_0 &= -\psi\log\delta+(1-\psi)r_f+\left(\tfrac{\rho-1}{\rho}\right)b_0+b_1\mu(1-\phi)+b_2\mu^2(1-\phi)^2+b_2\sigma_\eta^2+k \\
&\quad +0.5(1-\psi)\{\textstyle\sum_i a_{0i}\sigma_{ii}-\sum_i\sum_j a_{0i}a_{0j}\sigma_{ij}\}, \\
\tau_1 &= b_1\left(\tfrac{\rho\phi-1}{\rho}\right)+2b_2\mu\phi(1-\phi) \\
&\quad +(1-\psi)\{\textstyle\sum_i a_{0i}e_i+0.5\sum_i a_{1i}\sigma_{ii}-0.5\sum_i\sum_j(a_{0i}a_{1j}+a_{0j}a_{1i})\sigma_{ij}\}, \\
\tau_2 &= b_2\left(\tfrac{\rho\phi^2-1}{\rho}\right)+(1-\psi)\{\textstyle\sum_i a_{1i}e_i-0.5\sum_i\sum_j a_{1i}a_{1j}\sigma_{ij}\}.
\end{aligned}$$

Proof. Using the log-linearized Euler equation:

$$\mathbf{0}=\mathrm{E}_t\boldsymbol{r}_{t+1}-r_f-(1-\gamma)/(\psi-1)\boldsymbol{\sigma}_{c,t}-(\gamma-\psi^{-1})/(1-\psi^{-1})\boldsymbol{\sigma}_{p,t}+0.5\boldsymbol{\sigma}_i, \tag{14}$$

where $\boldsymbol{\sigma}_{c,t}\equiv[\sigma_{c1,t},\ldots,\sigma_{cn,t}]'$, $\boldsymbol{\sigma}_{p,t}\equiv[\sigma_{p1,t},\ldots,\sigma_{pn,t}]'$ and $\boldsymbol{\sigma}_i\equiv[\sigma_{11},\ldots,\sigma_{nn}]'$, as well as Lemmas 3 and 5 yields

$$\begin{aligned}
\mathrm{E}_t\Delta c_{t+1} &= \psi\log\delta+\psi E_t r_{p,t+1}+\tau_{p,t}, \\
&= (\psi\log\delta+\psi r_f+\psi p_0+\tau_0)+(\psi p_1+\tau_1)x_t+(\psi p_2+\tau_2)x_t^2, \\
&= c_0+c_1x_t+c_2x_t^2,
\end{aligned}$$

where the last line is obtained from Lemma 4. Hence,

$$\begin{aligned}
\tau_0 &= c_0-\psi\log\delta-\psi r_f-\psi p_0, \\
\tau_1 &= c_1-\psi p_1, \\
\tau_2 &= c_2-\psi p_2.
\end{aligned}$$

Substitute for c_0, c_1, c_2 from Lemma 4 and use the expressions for p_0, p_1, p_2 derived in Lemma 3 to solve for τ_0, τ_1, τ_2, as stated. ∎

A.2 Main results

Proposition 1 *The optimal shares are linear in the state variable, as postulated in candidate* (11).

Proof. Using Lemma 1, observe that:

$$\begin{aligned}
\sigma_{(\log\alpha_c)i,t} &= \mathrm{Cov}_t[r_{i,t+1},\log\alpha_{c,t+1}], \\
&= \mathrm{Cov}_t[u_{i,t+1},b_1(x_{t+1}-\mathrm{E}_tx_{t+1})+b_2(x_{t+1}^2-\mathrm{E}_tx_{t+1}^2)], \\
&= \{b_1+2b_2[\mu(1-\phi)+\phi x_t]\}\sigma_{i\eta}.
\end{aligned}$$

Substitute in the log-linearized Euler equation (14) to obtain:

$$\begin{aligned}\sum_j \alpha_{j,t}\sigma_{ij} &= \gamma^{-1}\left\{\mathrm{E}_t(r_{i,t+1}-r_f)+0.5\sigma_{ii}+\left(\tfrac{1-\gamma}{1-\psi}\right)\{b_1+2b_2[\mu(1-\phi)+\phi x_t]\}\sigma_{i\eta}\right\},\\ &= \gamma^{-1}\left\{0.5\sigma_{ii}+\left(\tfrac{1-\gamma}{1-\psi}\right)\sigma_{i\eta}[b_1+2b_2\mu(1-\phi)]\right\}+\gamma^{-1}\left\{e_i+\left(\tfrac{1-\gamma}{1-\psi}\right)\sigma_{i\eta}2b_2\phi\right\}x_t.\end{aligned}$$

∎

Proposition 2 *The optimal shares are as postulated in the candidate solutions* (10) and (11).

Proof. In Lemmas 5 and 6, equate the parameters τ_0, τ_1, τ_2 to obtain that:

$$\begin{aligned}0 \;=\;& \{-0.5(1-\gamma)/(1-\psi)[b_1^2+4b_1b_2\mu(1-\phi)+4b_2^2\mu^2(1-\phi)^2]-b_2\}\sigma_\eta^2\\ &+0.5(1-\gamma)/(1-\psi)b_2^2(\sigma_\eta^2)^2-(1-\gamma)\textstyle\sum_i[b_1+2b_2\mu(1-\phi)]a_{0i}\sigma_{\eta i}\\ &+0.5(1-\psi)\gamma\textstyle\sum_i\sum_j a_{0i}a_{0j}\sigma_{ij}-0.5(1-\psi)\sum_i a_{0i}\sigma_{ii}\\ &+\psi\log\delta-(1-\psi)r_f-b_0(\rho-1)/\rho-b_1\mu(1-\phi)-b_2\mu^2(1-\phi)^2-k\end{aligned}\tag{15}$$

in the case of τ_0, whereas

$$\begin{aligned}0 \;=\;& -(1-\gamma)/(1-\psi)[2b_1b_2\phi+4b_2^2\phi\mu(1-\phi)]\sigma_\eta^2\\ &-(1-\gamma)\textstyle\sum_i[b_1a_{1i}+2b_2\phi a_{0i}+2b_2\mu(1-\phi)a_{1i}]\sigma_{\eta i}\\ &+0.5(1-\psi)\gamma\textstyle\sum_i\sum_j(a_{0i}a_{1j}+a_{0j}a_{1i})\sigma_{ij}-b_1(\rho\phi-1)/\rho\\ &-2b_2\mu\phi(1-\phi)-(1-\psi)\textstyle\sum_i a_{0i}e_i-0.5(1-\psi)\sum_i a_{1i}\sigma_{ii}\end{aligned}\tag{16}$$

in the case of τ_1, while

$$\begin{aligned}0 \;=\;& -2(1-\gamma)/(1-\psi)b_2^2\phi^2\sigma_\eta^2-\textstyle\sum_i[2(1-\gamma)b_2\phi\sigma_{\eta i}+(1-\psi)e_i]a_{1i}\\ &+0.5(1-\psi)\gamma\textstyle\sum_i\sum_j a_{1i}a_{1j}\sigma_{ij}-b_2(\rho\phi^2-1)/\rho\end{aligned}\tag{17}$$

in the case of τ_2.

Next, solve (14) for the portfolio:

$$\begin{aligned}\boldsymbol{\alpha}_t &= \boldsymbol{\sigma}^{-1}\gamma^{-1}\{0.5\boldsymbol{\sigma}_i+(1-\gamma)/(1-\psi)\boldsymbol{\sigma}_\eta[b_1+2b_2\mu(1-\phi)]\}\\ &\quad+\boldsymbol{\sigma}^{-1}\gamma^{-1}\{\boldsymbol{e}+(1-\gamma)/(1-\psi)\boldsymbol{\sigma}_\eta 2b_2\phi\}x_t,\\ &= \boldsymbol{a}_0+\boldsymbol{a}_1x_t.\end{aligned}$$

Recall that $\boldsymbol{\sigma}_i \equiv [\sigma_{11},\ldots,\sigma_{nn}]'$ and $\boldsymbol{\sigma}_\eta \equiv [\sigma_{1\eta},\ldots,\sigma_{n\eta}]'$. Denote $\boldsymbol{\kappa} \equiv \boldsymbol{\sigma}^{-1}$ the inverse of the covariance matrix with $\boldsymbol{\kappa}_i$ the i^{th} column. Then, the loadings for optimal shares are:

$$a_{0i} = a_{00i}+a_{01i}b_1+a_{02i}b_2,\tag{18}$$

$$a_{1i} = a_{10i}+a_{12i}b_2,\tag{19}$$

where,

$$\begin{aligned}&a_{00i}\equiv 0.5\gamma^{-1}\boldsymbol{\kappa}_i'\boldsymbol{\sigma}_i; \quad a_{01i}\equiv(1-\gamma)/(1-\psi)\gamma^{-1}\boldsymbol{\kappa}_i'\boldsymbol{\sigma}_\eta; \quad a_{02i}\equiv(1-\gamma)/(1-\psi)\gamma^{-1}\boldsymbol{\kappa}_i'\boldsymbol{\sigma}_\eta 2\mu(1-\phi);\\ &a_{10i}\equiv\gamma^{-1}\boldsymbol{\kappa}_i'\boldsymbol{e}; \quad a_{12i}\equiv(1-\gamma)/(1-\psi)\gamma^{-1}\boldsymbol{\kappa}_i'\boldsymbol{\sigma}_\eta 2\phi;\end{aligned}$$

which establishes that the loadings $\boldsymbol{a}_0, \boldsymbol{a}_1$ are constant functions of the $\boldsymbol{b}$'s. Substitute the resulting expressions in (15–17) to obtain that:

$$0 = B^a+B_0^ab_0+B_1^ab_1+B_2^ab_2+B_{11}^ab_1^2+B_{12}^ab_1b_2+B_{22}^ab_2^2,\tag{20}$$

$$0 = B^b+B_1^bb_1+B_2^bb_2+B_{12}^bb_1b_2+B_{22}^bb_2^2,\tag{21}$$

$$0 = B^c+B_2^cb_2+B_{22}^cb_2^2,\tag{22}$$

where,

$$
\begin{aligned}
B^a &\equiv 0.5(1-\psi)\gamma \textstyle\sum_i \sum_j a_{00i}a_{00j}\sigma_{ij} - 0.5(1-\psi)\sum_i a_{00i}\sigma_{ii} \\
&\quad +\psi \log\delta - (1-\psi)r_f - k, \\
B_0^a &\equiv -(\rho-1)/\rho, \\
B_1^a &\equiv -(1-\gamma)\textstyle\sum_i a_{00i}\sigma_{\eta i} + 0.5(1-\psi)\gamma\sum_i\sum_j(a_{00j}a_{01i}+a_{00i}a_{01j})\sigma_{ij} \\
&\quad -0.5(1-\psi)\textstyle\sum_i a_{01i}\sigma_{ii} - \mu(1-\phi), \\
B_2^a &\equiv -\sigma_\eta^2 - 2(1-\gamma)\textstyle\sum_i \mu(1-\phi)a_{00i}\sigma_{\eta i} - \mu^2(1-\phi)^2 \\
&\quad +0.5(1-\psi)\gamma\textstyle\sum_i\sum_j(a_{00j}a_{02i}+a_{00i}a_{02j})\sigma_{ij} - 0.5(1-\psi)\sum_i a_{02i}\sigma_{ii}, \\
B_{11}^a &\equiv -0.5(1-\gamma)/(1-\psi)\sigma_\eta^2 - (1-\gamma)\textstyle\sum_i a_{01i}\sigma_{\eta i} + 0.5(1-\psi)\gamma\sum_i\sum_j a_{01i}a_{01j}\sigma_{ij}, \\
B_{12}^a &\equiv -2(1-\gamma)/(1-\psi)\mu(1-\phi)\sigma_\eta^2 - (1-\gamma)\textstyle\sum_i[2\mu(1-\phi)a_{01i}+a_{02i}]\sigma_{\eta i} \\
&\quad +0.5(1-\psi)\gamma\textstyle\sum_i\sum_j(a_{01i}a_{02j}+a_{01j}a_{02i})\sigma_{ij}, \\
B_{22}^a &\equiv -2(1-\gamma)/(1-\psi)\mu^2(1-\phi)^2\sigma_\eta^2 + 0.5(1-\gamma)/(1-\psi)(\sigma_\eta^2)^2 - 2(1-\gamma)\textstyle\sum_i \mu(1-\phi)a_{02i}\sigma_{\eta i} \\
&\quad +0.5(1-\psi)\gamma\textstyle\sum_i\sum_j a_{02i}a_{02j}\sigma_{ij}.
\end{aligned}
$$

Similarly,

$$
\begin{aligned}
B^b &\equiv -(1-\psi)\textstyle\sum_i e_i a_{00i} + 0.5(1-\psi)\gamma\sum_i\sum_j(a_{00i}a_{10j}+a_{00j}a_{10i})\sigma_{ij} \\
&\quad -0.5(1-\psi)\textstyle\sum_i a_{10i}\sigma_{ii}, \\
B_1^b &\equiv -(1-\gamma)\textstyle\sum_i a_{10i}\sigma_{\eta i} - (1-\psi)\sum_i e_i a_{01i} + 0.5(1-\psi)\gamma\sum_i\sum_j(a_{01i}a_{10j}+a_{01j}a_{10i})\sigma_{ij} \\
&\quad -(\rho\phi-1)/\rho, \\
B_2^b &\equiv -2(1-\gamma)\textstyle\sum_i \mu(1-\phi)a_{10i}\sigma_{\eta i} - 2(1-\gamma)\sum_i \phi a_{00i}\sigma_{\eta i} - (1-\psi)\sum_i e_i a_{02i} \\
&\quad +0.5(1-\psi)\gamma\textstyle\sum_i\sum_j(a_{00i}a_{12j}+a_{02i}a_{10j}+a_{00j}a_{12i}+a_{02j}a_{10i})\sigma_{ij} - 2\mu\phi(1-\phi) \\
&\quad -0.5(1-\psi)\textstyle\sum_i a_{12i}\sigma_{ii}, \\
B_{12}^b &\equiv -2(1-\gamma)/(1-\psi)\phi\sigma_\eta^2 - (1-\gamma)\textstyle\sum_i(a_{12i}+2\phi a_{01i})\sigma_{\eta i} \\
&\quad +0.5(1-\psi)\gamma\textstyle\sum_i\sum_j(a_{01i}a_{12j}+a_{01j}a_{12i})\sigma_{ij}, \\
B_{22}^b &\equiv -4(1-\gamma)/(1-\psi)\phi\mu(1-\phi)\sigma_\eta^2 - 2(1-\gamma)\textstyle\sum_i[\mu(1-\phi)a_{12i}+\phi a_{02i}]\sigma_{\eta i} \\
&\quad +0.5(1-\psi)\gamma\textstyle\sum_i\sum_j(a_{02i}a_{12j}+a_{02j}a_{12i})\sigma_{ij}.
\end{aligned}
$$

Finally,

$$
\begin{aligned}
B^c &\equiv -(1-\psi)\textstyle\sum_i e_i a_{10i} + 0.5(1-\psi)\gamma\sum_i\sum_j a_{10i}a_{10j}\sigma_{ij}, \\
B_2^c &\equiv -2(1-\gamma)\textstyle\sum_i \phi\sigma_{\eta i}a_{10i} - (1-\psi)\sum_i e_i a_{12i} \\
&\quad +0.5(1-\psi)\gamma\textstyle\sum_i\sum_j(a_{10i}a_{12j}+a_{10j}a_{12i})\sigma_{ij} - (\rho\phi^2-1)/\rho, \\
B_{22}^c &\equiv -2(1-\gamma)/(1-\psi)\phi^2\sigma_\eta^2 - 2(1-\gamma)\textstyle\sum_i \phi a_{12i}\sigma_{\eta i} + 0.5(1-\psi)\gamma\sum_i\sum_j a_{12i}a_{12j}\sigma_{ij}.
\end{aligned}
$$

The system (20), (21), and (22) defines a recursive quadratic system in b_0, b_1, b_2. To solve, use the following procedure:

1. From (22), solve for b_2 as:

$$b_2 = \{-B_2^c \pm [(B_2^c)^2 - 4B^c B_{22}^c]^{1/2}\}/(2B^c).$$

   Following Campbell and Viceira (1999), only the positive root is relevant.

2. Substitute for b_2 in (21) and solve for b_1 as:

$$b_1 = -(B^b + B_2^b b_2 + B_{22}^b b_2^2)/(B_1^b + B_{12}^b b_2).$$

3. Substitute b_1, b_2 in (20) and solve for b_0 as:

$$b_0 = -(B^a + B_1^a b_1 + B_2^a b_2 + B_{11}^a b_1^2 + B_{12}^a b_1 b_2 + B_{22}^a b_2^2)/B_0^a.$$

4. Finally, substitute b_0, b_1, b_2 in the expressions (18) and (19) and solve for a_{0i}, a_{1i}.

It follows immediately that the loadings are constant as postulated. ∎

B Maximum Likelihood Procedure

The multivariate stochastic excess-return process (7–8) is rewritten as:

$$\boldsymbol{r}_t^e = \begin{bmatrix} \frac{(\boldsymbol{e}\sigma_\eta)}{(1-\phi^2)^{1/2}} & \frac{(\boldsymbol{e}\sigma_\eta)}{\lambda} \end{bmatrix} \begin{bmatrix} (1-\phi^2)^{1/2}\left(\frac{x_{t-1}}{\sigma_\eta}\right) \\ \left(\frac{\eta_t}{\sigma_\eta}\right) \end{bmatrix} + \boldsymbol{\nu}_t, \tag{23}$$

$$\begin{bmatrix} (1-\phi^2)^{1/2}\left(\frac{x_{t-1}}{\sigma_\eta}\right) \\ \left(\frac{\eta_t}{\sigma_\eta}\right) \end{bmatrix} = \begin{bmatrix} (1-\phi)(1-\phi^2)^{1/2}\left(\frac{\mu}{\sigma_\eta}\right) \\ 0 \end{bmatrix} + \begin{bmatrix} \phi & (1-\phi^2)^{1/2} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} (1-\phi^2)^{1/2}\left(\frac{x_{t-2}}{\sigma_\eta}\right) \\ \left(\frac{\eta_{t-1}}{\sigma_\eta}\right) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}\left(\frac{\eta_t}{\sigma_\eta}\right), \tag{24}$$

or, more compactly as

$$\boldsymbol{r}_t^e = \boldsymbol{\zeta}\boldsymbol{f}_t + \boldsymbol{\nu}_t, \tag{25}$$

$$\boldsymbol{f}_t = \boldsymbol{\xi}_0 + \boldsymbol{\xi}_1\boldsymbol{f}_{t-1} + \boldsymbol{\xi}_2(\eta_t/\sigma_\eta). \tag{26}$$

The expression (25) corresponds to a measurement equation that relates the observable excess returns $\boldsymbol{r}_t^e$ to the unobservable common factors $\boldsymbol{f}_t$. The expression (26) is the transition equation which describes the stochastic properties of the common factors. As is standard practice, these equations imply orthogonal factors with unconditional variances equal to one. Furthermore, the sensitivity parameter e_{inc} of the corporate-equity excess return $r_{inc,t}^e$ to the state variable x_t is fixed to unity. This normalization ensures the identification of all other individual parameters involved in (25–26), rather than simply the products $(\boldsymbol{e}\sigma_\eta)$ and the ratio (μ/σ_η). The normalization also guarantees the identification of the sign of x_t.

The free parameters of the process (25)–(26) are estimated by applying a maximum-likelihood procedure. This procedure first applies the Kalman filter to yield:

$$\boldsymbol{f}_{t/t-1} \equiv E_{t-1}\boldsymbol{f}_t = \boldsymbol{\xi}_0 + \boldsymbol{\xi}_1\boldsymbol{f}_{t-1/t-1}, \tag{27}$$

$$\boldsymbol{\Lambda}_{t/t-1} \equiv E_{t-1}(\boldsymbol{f}_t\boldsymbol{f}_t') = \boldsymbol{\xi}_1\boldsymbol{\Lambda}_{t-1/t-1}\boldsymbol{\xi}_1' + \boldsymbol{\xi}_2\boldsymbol{\xi}_2', \tag{28}$$

$$\boldsymbol{u}_{t/t-1} \equiv (\boldsymbol{r}_t^e - E_{t-1}\boldsymbol{r}_t^e) = \boldsymbol{r}_t^e - \boldsymbol{\zeta}\boldsymbol{f}_{t/t-1}, \tag{29}$$

$$\boldsymbol{\Sigma}_{t/t-1} \equiv E_{t-1}(\boldsymbol{u}_t\boldsymbol{u}_t') = \boldsymbol{\zeta}\boldsymbol{\Lambda}_{t/t-1}\boldsymbol{\zeta}' + \boldsymbol{\omega}, \tag{30}$$

$$\boldsymbol{f}_{t/t} \equiv E_t(\boldsymbol{f}_t) = \boldsymbol{f}_{t/t-1} + \boldsymbol{\Lambda}_{t/t-1}\boldsymbol{\zeta}'\boldsymbol{\Sigma}_{t/t-1}^{-1}\boldsymbol{u}_{t/t-1}, \tag{31}$$

$$\boldsymbol{\Lambda}_{t/t} \equiv E_t(\boldsymbol{f}_t\boldsymbol{f}_t') = \boldsymbol{\Lambda}_{t/t-1} - \boldsymbol{\Lambda}_{t/t-1}\boldsymbol{\zeta}'\boldsymbol{\Sigma}_{t/t-1}^{-1}\boldsymbol{\zeta}\boldsymbol{\Lambda}_{t/t-1}, \tag{32}$$

where $\boldsymbol{\omega} = E_{t-1}(\boldsymbol{\nu}_t\boldsymbol{\nu}_t')$. The prediction equations (27) and (28) establish the forecasts of the current variables given lagged information. Equations (29) and (30) provides the prediction error and its covariance matrix. The updating equations (31) and (32) revise the forecasts given the availability of current information. These updates provide the best (in the conditional mean square error sense) estimates of the unspecified factors. The formulae (27)–(32) are evaluated recursively for $t = 1, \ldots, T$ by initializing $\boldsymbol{f}_{0/0} = (\boldsymbol{I} - \boldsymbol{\xi}_1)^{-1}\boldsymbol{\xi}_0$ and $\boldsymbol{\Lambda}_{0/0} = \boldsymbol{I}$ from the unconditional moments, and by giving values to the parameters $\boldsymbol{\Theta}$ – where $\boldsymbol{\Theta}$ comprises all the unconstrained elements of $\boldsymbol{e}$ and $\boldsymbol{\omega}$, as well as μ, ϕ, λ, and σ_η^2.

The Kalman filter is used to construct the approximate log-likelihood of the sample (ignoring the constant term) associated with conditionally Gaussian unanticipated components:

$$L(\boldsymbol{u}, \boldsymbol{\Theta}) = -\frac{1}{2}\sum_{t=1}^{T}\log|\boldsymbol{\Sigma}_{t/t-1}| - \frac{1}{2}\sum_{t=1}^{T}\boldsymbol{u}_{t/t-1}'\boldsymbol{\Sigma}_{t/t-1}^{-1}\boldsymbol{u}_{t/t-1}. \tag{33}$$

Then, the estimates are derived by maximizing (33) over the parameters $\boldsymbol{\Theta}$. This exercise is performed by using the DFP algorithm in GQOPT.

References

Board of Governors (1999) ‘Flow of funds accounts of the United States.’ Statistical Release Z.1, Historical data, Federal Reserve Bank

Brennan, Michael J., and Yihong Xia (2000) ‘Stochastic interest rates and the bond-stock mix.’ mimeo

Campbell, John Y. (1993) ‘Intertemporal asset pricing without consumption data.’ *American Economic Review* 83(3), 487–512

— (1996) ‘Understanding risk and return.’ *Journal of Political Economy* 104(2), 298–345

Campbell, John Y., and Angus Deaton (1989) ‘Why is consumption so smooth?’ *Review of Economic Studies* 56(3), 357–373

Campbell, John Y., and Gregory N. Mankiw (1989) ‘Consumption, income and interest rates: Reinterpreting the time series evidence.’ In *NBER Macroeconomics Annual: 1989,* ed. Olivier J. Blanchard and Stanley Fischer Econometric Society Monographs (Cambridge, Mass.: MIT Press) pp. 185–216

Campbell, John Y., and Hyeng Keun Koo (1997) ‘A comparison of numerical and analytical approximate solutions to an intertemporal consumption choice problem.’ *Journal of Economic Dynamics and Control* 21(2–3), 273–295

Campbell, John Y., and Luis M. Viceira (1999) ‘Consumption and portfolio decisions when expected returns are time varying.’ *Quarterly Journal of Economics* 114(2), 433–495

Campbell, John Y., João Cocco, Francisco Gomes, Pascal J. Maenhout, and Luis M. Viceira (1998) ‘Stock market mean reversion and the optimal equity allocation of a long-lived investor.’ unpublished paper, Harvard University

Campbell, John Y., Yeung L. Chan, and Luis M. Viceira (1999) ‘A multivariate model of strategic asset allocation.’ unpublished paper, Harvard University, September

Canner, Niko, N. Gregory Mankiw, and David N. Weil (1997) ‘An asset allocation puzzle.’ *American Economic Review* 87(1), 191–191

Chou, Ray Y., Robert F. Engle, and Alex Kane (1992) ‘Measuring risk aversion from excess returns on a stock index.’ *Journal of Econometrics* 52(1–2), 201–224

Deaton, Angus (1987) ‘Life-cycle models of consumption: Is the evidence consistent with the theory?’ In *Advances in Econometrics: Fifth World Congress,* ed. Truman B. Bewley, vol. 2 (Cambridge, UK: Cambridge University Press) pp. 121–148

Eisner, Robert (1989) *The Total Incomes System of Accounts* (Chicago and London: The University of Chicago Press)

Elton, Edwin J., and Martin J. Gruber (2000) ‘The rationality of asset allocation recommendations.’ *Journal of Financial and Quantitative Analysis* 35(1), 27–41

Epstein, Larry G., and Stanley E. Zin (1989) ‘Substitution, risk aversion and the temporal behavior of consumption and asset returns: A theoretical framework.’ *Econometrica* 57(4), 937–969

Fabozzi, Frank J. (1999) *Investment Management,* second ed. (Upper Saddle River, NJ: Prentice Hall)

Fama, Eugene F., and William G. Schwert (1977) ‘Asset returns and inflation.’ *Journal of Financial Economics* 5(2), 115–146

Flavin, Marjorie, and Takashi Yamashita (1998) ‘Owner-occupied housing and the composition of the household portfolio over the life cycle.’ discussion paper 98-02, Department of Economics, University of California, San Diego, January

Giovannini, Alberto, and Philippe Weil (1989) ‘Risk-aversion and inter-temporal substitution in the capital asset pricing model.’ Working Paper 2824, National Bureau of Economic Research, January

Goetzmann, William Nelson (1993) ‘The single family home in the investment portfolio.’ *Journal of Real Estate Finance and Economics* 6, 201–222

Haliassos, Michael, and Carol C. Bertaut (1995) ‘Why do so few hold stocks?’ *Economic Journal* 105(432), 1110–1129

Hall, Robert E. (1988) ‘Intertemporal substitution in consumption.’ *Journal of Political Economy* 96(2), 339–357

Heaton, John C., and Deborah J. Lucas (1997) ‘Savings behavior and portfolio choices: Which risks matter.’ mimeo, Northwestern University and NBER

— (2000a) ‘Portfolio choice and asset prices: The importance of entrepreneurial risk.’ *Journal of Finance* 55(3), 1163–1197

— (2000b) ‘Portfolio choice in the presence of background risk.’ *Economic Journal* 110, 1–26

Ibbotson, Roger G., and Gary P. Brinson (1987) *Investment markets* (New York, NY: McGraw-Hill)

Jagannathan, Ravi, and Zhenyu Wang (1993) ‘The CAPM is alive and well.’ Staff Report 165, Federal Reserve Bank of Minneapolis, November

Jagannathan, Ravi, and Zhiwu Wang (1996) ‘The conditional CAPM and the cross-section of expected returns.’ *Journal of Finance* 51, 3–53

Jorgenson, Dale W., and Barbara M. Fraumeni (1989) ‘The accumulation of human and nonhuman capital, 1948–84.’ In *The Measurement of Savings, Investment and Wealth,* ed. Robert E. Lipsey and Helen Stone Rice, vol. 52 of *Studies in Income and Wealth* National Bureau of Economic Research The University of Chicago Press Chicago and London pp. 227–285

Jorion, Philippe, and Alberto Giovannini (1993) ‘Time-series tests of a non-expected utility model of asset pricing.’ *European Economic Review* 37(5), 1083–1100

Kendrick, John W. (1976) *The Formation of Stocks of Total Capital* number 100. In ‘General Series.’ (New York: National Bureau of Economic Research)

Mankiw, Gregory N., and Stephen P. Zeldes (1991) ‘The consumption of stockholders and non-stockholders.’ *Journal of Financial Economics* 29(1), 97–112

Mehra, Rajnish, and Edward C. Prescott (1985) ‘The equity premium: A puzzle.’ *Journal of Monetary Economics* 15(2), 145–161

Merton, Robert C. (1971) ‘Optimum consumption and portfolio rules in a continuous-time model.’ *Journal of Economic Theory* 3(4), 373–413

— (1973) ‘An inter-temporal capital asset pricing model.’ *Econometrica* 41, 867–888

Normandin, Michel, and Pascal St-Amour (1998) ‘Substitution, risk aversion, taste shocks and equity premia.’ *Journal of Applied Econometrics* 13(3), 265–281

Roll, Richard (1977) ‘A critique of the asset pricing theory’s tests: Part 1: On past and potential testability of the theory.’ *Journal of Financial Economics* 4(2), 129–176

Samuelson, Paul A. (1969) ‘Lifetime portfolio selection by dynamic stochastic programming.’ *Review of Economic and Statistics* 51(3), 239–246

Shiller, Robert J. (1995) ‘Aggregate income risks and hedging mechanisms.’ *Quarterly Review of Economics and Finance* 35(2), 119–152

Weil, Philippe (1989) ‘The equity premium puzzle and the risk-free rate puzzle.’ *Journal of Monetary Economics* 24(3), 401–421

— (1990) ‘Non-expected utility in macroeconomics.’ *Quarterly Journal of Economics* 105(1), 29–42

Tables

Table 1: Comparisons with Other Measures of Total Wealth

A. Wealth-income ratio				
	mean	std. dev.	min	max
W_{rc}/Y	6.50	0.61	5.38	7.72
W_{bc}/Y	30.99	2.46	27.59	36.24
$W_{pv,1}/Y$	34.85	1.03	32.78	36.06
$W_{pv,2}/Y$	59.52	2.43	53.75	63.60

B. Portfolio excess returns				
	mean	std. dev.	min	max
r^e_{rc}	12.08	2.84	10.28	26.81
r^e_{bc}	2.88	1.87	-0.95	8.64
$r^e_{pv,1}$	3.21	2.94	-3.43	12.86
$r^e_{pv,2}$	2.34	2.92	-5.91	6.33

Panel A, sample covers the 1949-1981 period. Panel B, sample covers the 1950-1981 period. Excess portfolio returns on total wealth is $r_t = [W_t/(W_{t-1} - C_{t-1}) - 1]$ minus the riskfree rate.

rc Replacement cost method. Total capital, year-end totals (Eisner 1989, table 13, pp. 263-66).

bc Budget constraint method. Total wealth and portfolio return are constructed recursively from equations (1) and (2).

$pv, 1$ Present value of method. Total human wealth plus total tangible and total financial assets. Human wealth is the expected net present value of labor income (compensation of employees, wages and salaries), which is assumed to follow an first-order auto-regressive process. Rate of discounting is fixed at 3.5%.

$pv, 2$ Present value method. Incomes related to human wealth include both labor income and the value of nonmarket activities. Full private national wealth (Jorgenson and Fraumeni 1989, table 5.32, p. 271).

Table 2: Descriptive Statistics: Empirical Shares

A. Means, standard deviations and extrema (in %)				
	mean	std. dev.	min	max
α_c	1.85	0.19	1.59	2.19
α_{dep}	1.69	0.31	1.19	2.21
α_{res}	1.55	0.82	0.71	4.06
α_{inc}	1.62	0.61	0.82	3.10
α_{ninc}	2.32	0.34	1.70	3.01
α_{home}	2.18	0.31	1.71	2.90

B. Correlations (in %)

	α_{dep}	α_{res}	α_{inc}	α_{ninc}	α_{home}
α_c	78.45	89.91	-14.26	-61.13	67.73
α_{dep}		48.89	-24.97	-34.19	71.73
α_{res}			9.99	-73.56	49.85
α_{inc}				-47.55	-48.73
α_{ninc}					-22.81

Note: Panels A and B, α_c is the consumption share: $\alpha_{c,t} \equiv C_t/W_t$. α_{dep}, α_{res}, α_{inc}, α_{ninc} and α_{home} are the portfolio shares associated with deposits, reserves, as well as corporate, noncorporate and home equities, i.e. $\alpha_{i,t} \equiv W_{i,t}/W_t$. The sample covers the 1945–1998 period.

Table 3: Descriptive Statistics: Empirical Excess Returns

A. Means, standard deviations and extrema (in %)

	mean	std. dev.	min	max
r_p^e	2.66	3.15	-2.24	15.18
r_{dep}^e	2.97	3.84	-4.79	11.25
r_{res}^e	6.01	3.90	-4.43	15.27
r_{inc}^e	5.16	17.18	-40.94	38.25
r_{ninc}^e	1.80	4.83	-6.91	15.50
r_{home}^e	3.25	5.43	-9.15	16.41

B. Correlations (in %)

	r_{dep}^e	r_{res}^e	r_{inc}^e	r_{ninc}^e	r_{home}^e
r_p^e	31.33	22.69	-1.08	73.75	59.47
r_{dep}^e		23.54	20.77	44.00	45.53
r_{res}^e			73.17	12.09	29.85
r_{inc}^e				-5.54	10.04
r_{ninc}^e					70.91
β_{ip}	0.38	0.28	-0.06	1.13	1.03

Note: Panels A and B, excess returns are the returns on the total wealth portfolio:

$$r_{p,t+1} = Y_t/(W_t - Y_t),$$

and on individual assets:

$$r_{i,t+1} = W_{i,t+1}/[\alpha_{i,t}(W_t - C_t)] - 1,$$

minus the riskfree rate. r_p^e is the excess return on total wealth portfolio. r_{dep}^e, r_{res}^e, r_{inc}^e, r_{ninc}^e and r_{home}^e are the excess returns on deposits, reserves, as well as corporate, noncorporate and home equities. β_{ip} is total wealth portfolio beta of asset i. The sample covers the 1946–1998 period.

Table 4: Maximum Likelihood Estimates

A. Multivariate stochastic process for excess returns

$$\boldsymbol{r}_t^e = \boldsymbol{e}x_{t-1} + [(\boldsymbol{e}/\lambda)\eta_t + \boldsymbol{\nu}_t], \qquad \boldsymbol{\omega} = E[\boldsymbol{\nu}_t\boldsymbol{\nu}_t'],$$

$$x_t = \mu(1-\phi) + \phi x_{t-1} + \eta_t, \qquad \sigma_\eta^2 = E[\eta_t^2], \quad \text{where}$$

$$\boldsymbol{r}_t^e = [r_{dep,t}^e, \quad r_{res,t}^e, \quad r_{inc,t}^e, \quad r_{ninc,t}^e, \quad r_{home,t}^e]',$$

$$\boldsymbol{e} = \begin{bmatrix} 0.770 & 1.030 & 1.000 & 0.761 & 1.061 \\ (68.14) & (68.09) & (-) & (68.24) & (68.20) \end{bmatrix}',$$

$$\text{Diag}(\boldsymbol{\omega}^{1/2}) = \begin{bmatrix} 0.029 & 0.050 & 0.171 & 0.034 & 0.034 \\ (6.64) & (6.91) & (10.04) & (6.22) & (5.39) \end{bmatrix}',$$

$$\begin{array}{cccccccccccc} \lambda & = & 1.459, & \mu & = & 0.038, & \phi & = & 0.456, & (\sigma_\eta^2)^{1/2} & = & 0.027. \\ & & (2.02) & & & (0.002) & & & (1.86) & & & (1.83) \end{array}$$

B. Derived conditional second moments (CSM)

$$\begin{array}{rcl} \boldsymbol{\sigma} & = & (\boldsymbol{e}/\lambda)\sigma_\eta^2(\boldsymbol{e}/\lambda)' + \boldsymbol{\omega}, \\ \boldsymbol{\sigma}_\eta & = & (\boldsymbol{e}/\lambda)\sigma_\eta^2, \\ \text{Corr}_\eta & = & \boldsymbol{\sigma}_s^{-1}\boldsymbol{\sigma}_\eta(\sigma_\eta^2)^{-1/2}, \\ SR_t & \equiv & \boldsymbol{\sigma}_s^{-1}\boldsymbol{e}x_t, \quad \text{where } \boldsymbol{\sigma}_s^2 \equiv \text{Diag}(\boldsymbol{\sigma}), \text{ and} \end{array}$$

$$\boldsymbol{\sigma} = \begin{bmatrix} 1.048e-03 & & & & \\ (2.53) & & & & \\ 2.683e-04 & 2.814e-03 & & & \\ (0.62) & (2.98) & & & \\ 2.604e-04 & 3.487e-04 & 0.030 & & \\ (0.63) & (0.63) & (5.05) & & \\ 1.981e-04 & 2.653e-04 & 2.575e-04 & 1.363e-03 & \\ (0.62) & (0.62) & (0.63) & (2.85) & \\ 2.762e-04 & 3.697e-04 & 3.589e-04 & 2.730e-04 & 1.520e-03 \\ (0.62) & (0.62) & (0.63) & (0.62) & (2.06) \end{bmatrix},$$

$$\boldsymbol{\sigma}_\eta = \begin{bmatrix} 3.791e-04 & 5.075e-04 & 4.926e-04 & 3.748e-04 & 5.224e-04 \\ (0.78) & (0.78) & (0.79) & (0.78) & (0.78) \end{bmatrix}',$$

$$\text{Corr}_\eta = \begin{bmatrix} 0.437 & 0.357 & 0.107 & 0.379 & 0.501 \\ (1.76) & (1.56) & (0.43) & (1.58) & (1.99) \end{bmatrix}',$$

$$SR_t = \begin{bmatrix} 23.785 & 19.417 & 5.774 & 20.613 & 27.214 \\ (5.32) & (6.30) & (26.87) & (5.88) & (4.32) \end{bmatrix}' x_t.$$

Note: Entries are the maximum likelihood estimates (t-statistics), except for e_{inc} which is normalized at unity to identify the other parameters, and the sign of the state variable – see Appendix B for details. For Panel B, the t-statistics use the variance of each CSM, which is computed as $\boldsymbol{D}'\boldsymbol{\sigma}\boldsymbol{D}$ – where $\boldsymbol{D}$ is the vector of analytical derivatives of each CSM with respect to the parameters of the multivariate stochastic process for excess returns. The sample covers the 1946–1998 period.

Table 5: Means of Predicted and Empirical Shares

A. Predicted shares							
Preferences		$\overline{\alpha}_c$	$\overline{\alpha}_{dep}$	$\overline{\alpha}_{res}$	$\overline{\alpha}_{inc}$	$\overline{\alpha}_{ninc}$	$\overline{\alpha}_{home}$
$\gamma = 1$							
	ψ =1	4.00***	1844*	881*	118***	1337**	1893**
	1/4	72.62***	1844*	881*	118***	1337**	1893**
	1/10	78.71***	1844*	881*	118***	1337**	1893**
	1/40	81.21***	1844*	881*	118***	1337**	1893**
$\gamma = 4$							
	ψ =1	4.00***	302*	147*	23***	220**	310**
	1/4	12.60***	311*	151*	24***	227**	319**
	1/10	14.57***	313*	152*	24***	228**	322**
	1/40	15.60***	314*	152*	24***	229**	323**
$\gamma = 10$							
	ψ =1	4.00***	116*	56*	9.2***	84**	119**
	1/4	5.32***	116*	57*	9.2***	85**	119**
	1/10	5.60***	116*	57*	9.2***	85**	120**
	1/40	5.70***	116*	57*	9.2***	85**	120**
$\gamma = 40$							
	ψ =1	4.00***	28	14	2.3	21*	29**
	1/4	2.45**	28	14	2.3	21*	29**
	1/10	2.14	28	14	2.3	21*	29**
	1/40	1.98	28	14	2.3	21*	29**

B. Empirical shares							
		1.85	1.69	1.54	1.62	2.32	2.18

Note: A. Predicted shares:

$$\begin{aligned} \log \alpha_{c,t} &= b_0 + b_1 x_t + b_2 x_t^2, \\ \boldsymbol{\alpha}_t &= \boldsymbol{a}_0 + \boldsymbol{a}_1 x_t, \end{aligned}$$

where $\overline{\alpha}_c = \exp[\mathrm{E}(\log \alpha_{c,t})]$ is defined as the mean of consumption share. Portfolio shares $\overline{\alpha}_{dep}$, $\overline{\alpha}_{res}$, $\overline{\alpha}_{inc}$, $\overline{\alpha}_{ninc}$, and $\overline{\alpha}_{home}$ associated with deposits, reserves, as well as corporate, noncorporate, and home equities are evaluated at the mean of x_t. B. Sample estimates of the average empirical shares. All values in percentage. *, **, *** indicate that the difference between the predicted and empirical means is significant at the 15, 10 and 5 percent levels respectively. These tests are performed by using the variance of the difference, which is computed as $\boldsymbol{D}'\boldsymbol{\sigma}\boldsymbol{D}$ – where $\boldsymbol{D}$ is the vector of numerical derivatives of the difference with respect to the parameters of the multivariate stochastic process for excess returns. The sample covers the 1945–1998 period.

Table 6: Predicted Parameters of the Decision Rules and Empirical Estimates of the Unrestricted Reduced Forms

Preferences		b_0	b_1	b_2	$a_{0,dep}$	$a_{1,dep}$	$a_{0,res}$	$a_{1,res}$	$a_{0,inc}$	$a_{1,inc}$	$a_{0,ninc}$	$a_{1,ninc}$	$a_{0,home}$	$a_{1,home}$
A. Predicted parameters														
$\gamma = 1$														
	ψ =1	-3.22***	0***	0***	0.18	479.00	0.37***	221.33*	0.49***	18.11***	0.27***	343.64	0.21	490.94
	1/4	-0.73***	1.24***	152.03	0.18	479.00	0.37***	221.33*	0.49***	18.11***	0.27***	343.64	0.21	490.94
	1/10	-0.61***	0.93***	140.03	0.18	479.00	0.37***	221.33*	0.49***	18.11***	0.27***	343.64	0.21	490.94
	1/40	-0.55***	0.81***	133.12	0.18	479.00	0.37***	221.33*	0.49***	18.11***	0.27***	343.64	0.21	490.94
$\gamma = 4$														
	ψ =1	-3.22***	0***	0***	-0.77	99.31	-0.28	45.89*	0.09***	3.75***	-0.51***	71.25	-0.78***	101.79
	1/4	-2.27***	-0.41***	88.71	-0.73	100.66	-0.27	46.51*	0.09***	3.81***	-0.49***	72.22	-0.74***	103.17
	1/10	-2.16***	-0.46***	104.74	-0.72	100.99	-0.26	46.66*	0.09***	3.82***	-0.48***	72.45	-0.73***	103.50
	1/40	-2.11***	-0.48***	112.45	-0.72	101.15	-0.26	46.74*	0.09***	3.82***	-0.48***	72.57	-0.73***	103.68
$\gamma = 10$														
	ψ =1	-3.22***	0***	0***	-0.31	38.72	-0.12	17.89*	0.04**	1.46***	-0.22***	27.78	-0.32***	39.68
	1/4	-3.01***	-0.28***	35.27	-0.32	38.80	-0.12	17.93*	0.04**	1.47***	-0.21***	27.84	-0.32***	39.77
	1/10	-2.97***	-0.33***	42.24	-0.32	38.82	-0.12	17.94*	0.04**	1.47***	-0.21***	27.85	-0.32***	39.79
	1/40	-2.95***	-0.36***	42.71	-0.32	38.83	-0.12	17.94*	0.04**	1.47***	-0.21***	27.86	-0.32***	39.80
$\gamma = 40$														
	ψ =1	-3.22***	0***	0***	-0.08*	9.56	-0.03*	4.42*	9.00e-03***	0.36***	-0.05***	6.86	-0.08***	9.80
	1/4	-3.73	-0.08***	8.72***	-0.08*	9.54	-0.03*	4.41*	9.00e-03***	0.36***	-0.06***	6.84	-0.08***	9.78
	1/10	-3.87	-0.10***	10.49	-0.08*	9.53	-0.03*	4.41*	9.00e-03***	0.36***	-0.06***	6.84	-0.08***	9.77
	1/40	-3.94	-0.11***	11.37	-0.08*	9.53	-0.03*	4.40*	9.00e-03***	0.36***	-0.06***	6.84	-0.08***	9.77
B. Empirical estimates														
		-3.91	-3.63	24.43	0.018	-0.027	0.020	-0.130	0.016	0.005	0.020	0.076	0.023	-0.044

Note: A. Predicted parameters of the decision rules:

$$\begin{aligned} \log \alpha_{c,t} &= b_0 + b_1 x_t + b_2 x_t^2, \\ \boldsymbol{\alpha}_t &= \boldsymbol{a}_0 + \boldsymbol{a}_1 x_t. \end{aligned}$$

B. Ordinary least squares estimates of the associated unrestricted reduced forms. *, **, *** indicate that the difference between the predicted and empirical coefficients is significant at the 15, 10 and 5 percent levels respectively. These tests are performed by using the variance of the difference, which is computed as $\boldsymbol{D}'\boldsymbol{\sigma}\boldsymbol{D}$ – where $\boldsymbol{D}$ is the vector of numerical derivatives of the difference with respect to the parameters of the multivariate stochastic process for excess returns. The sample covers the 1945–1998 period.

Figure Legends

1. Total wealth of households and nonprofit organizations held in corporate equity (Board of Governors 1999, series FL153064105) divided by aggregate consumption of nondurables and services (source NIPA). The sample covers the 1945–1998 period.

2. The sample covers the 1945–1998 period.

3. α_{dep}, α_{res}, α_{inc}, α_{ninc} and α_{home} are the empirical portfolio shares associated with deposits, reserves, as well as corporate, noncorporate and home equities. The sample covers the 1945–1998 period.

4. The solid lines are the excess returns. The dashed line is the Kalman-filter estimate of the state variable. The sample covers the 1946–1998 period.

5. The solid line is the empirical excess return on corporate equity. The dashed line is the historical excess return on the S&P500 stock-market index. The sample covers the 1946–1998 period.

6. The solid line is the Kalman-filter estimate of the state variable. The dashed line is the historical dividend yield on the S&P500 stock-market index. The sample covers the 1945–1998 period.

7. α_{dep}, α_{res}, α_{inc}, α_{ninc} and α_{home} are the predicted portfolio shares associated with deposits, reserves, as well as corporate, noncorporate and home equities. The shares are evaluated for different values of $\gamma \in [1, 40]$, with $\psi = 1/40$, and x_t set to its sample mean. The sample covers the 1945–1998 period.

Figures

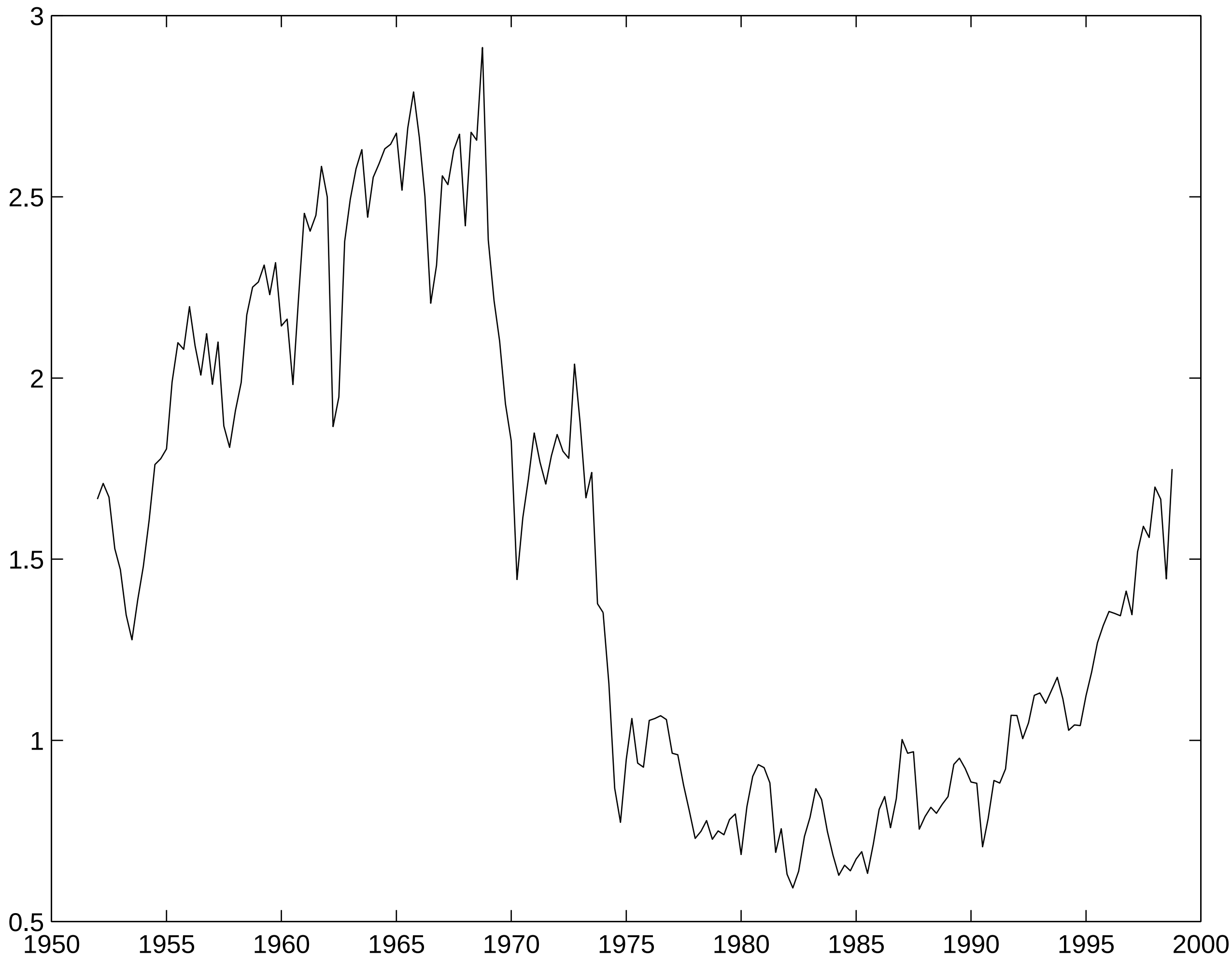

Figure 1: Corporate Equity Relative to Consumption

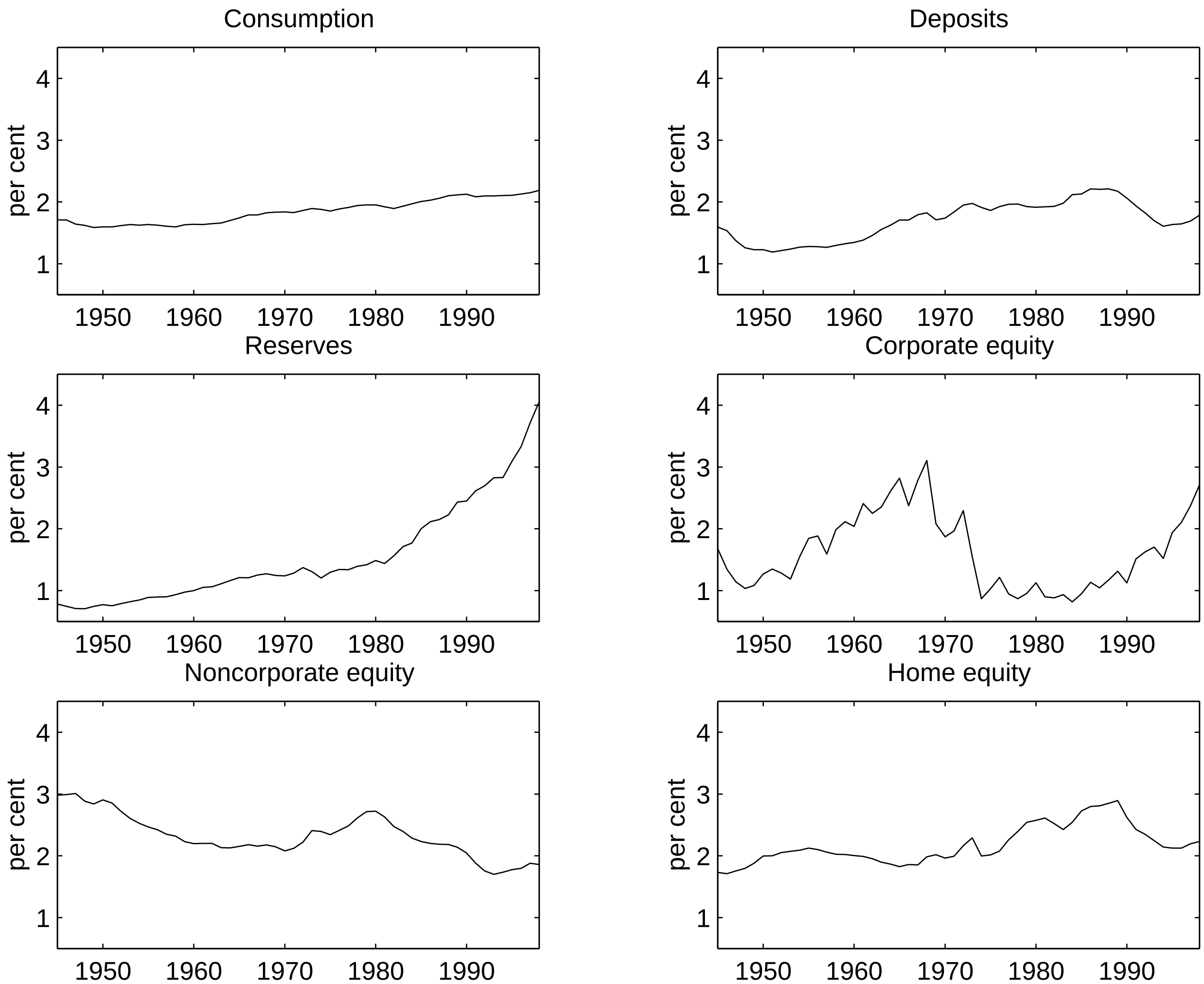

Figure 2: Empirical Consumption and Portfolio Shares

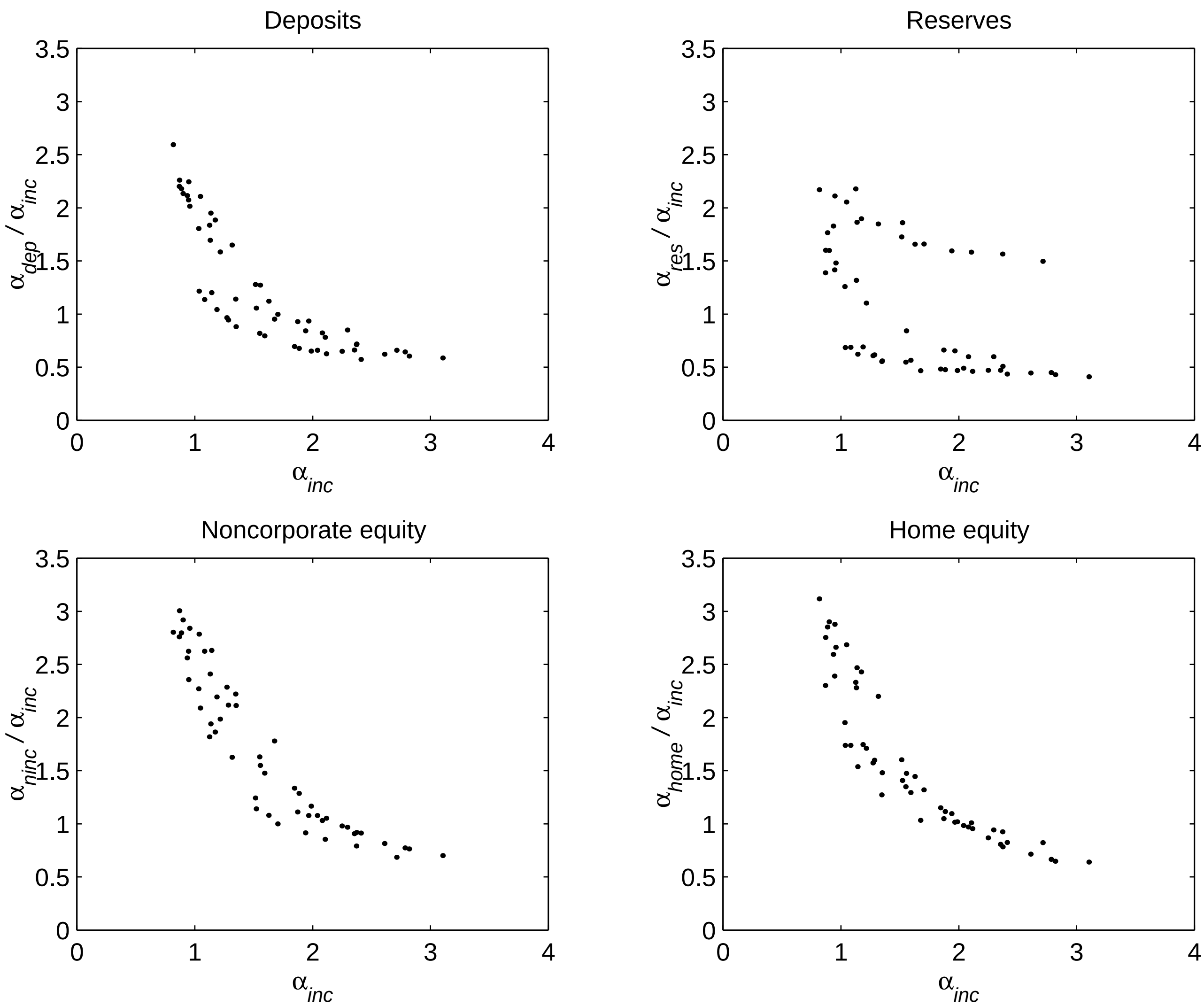

Figure 3: Empirical Composition of Risky Assets Relative to Corporate Equity

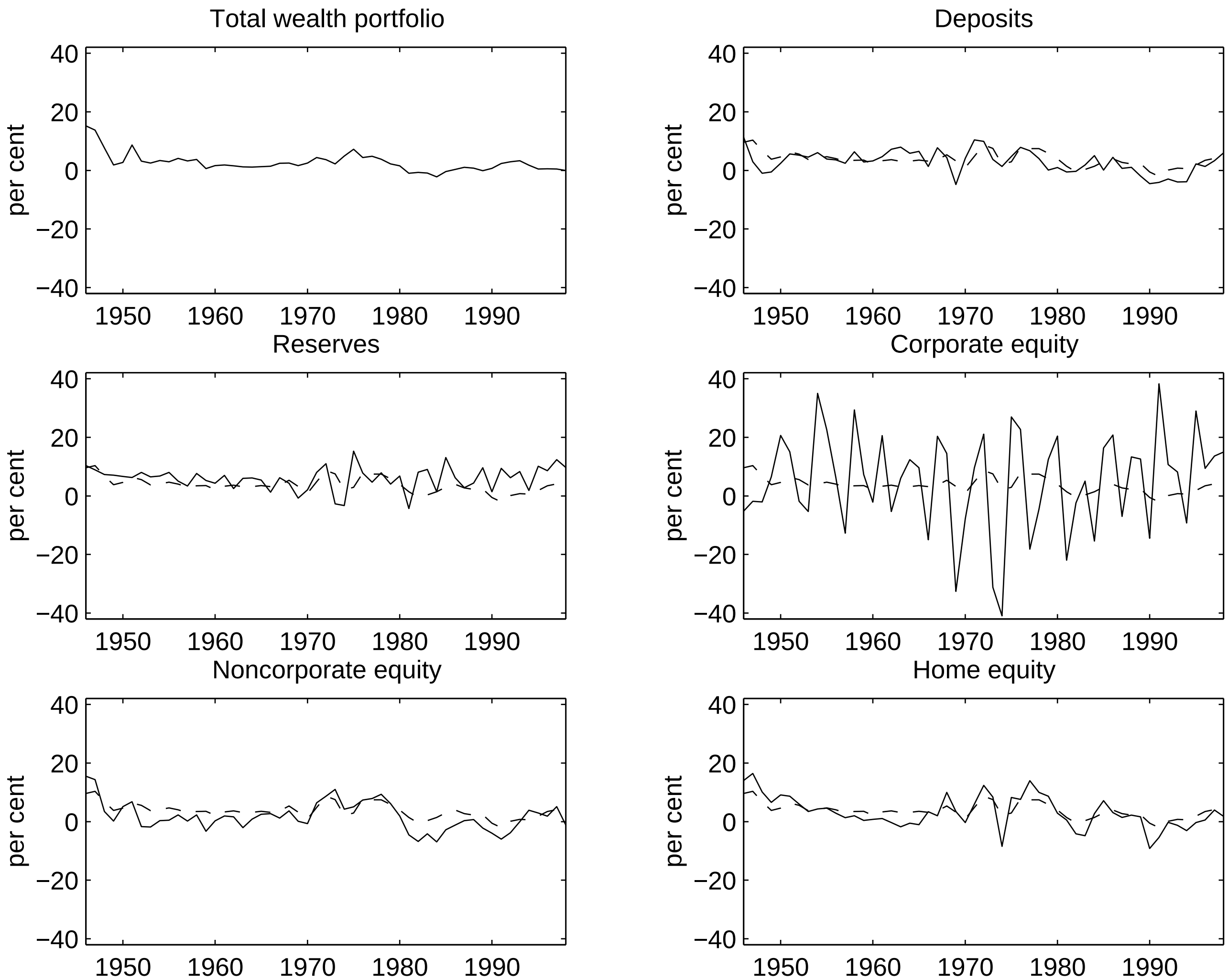

Figure 4: Empirical Total Wealth Portfolio and Individual Excess Returns

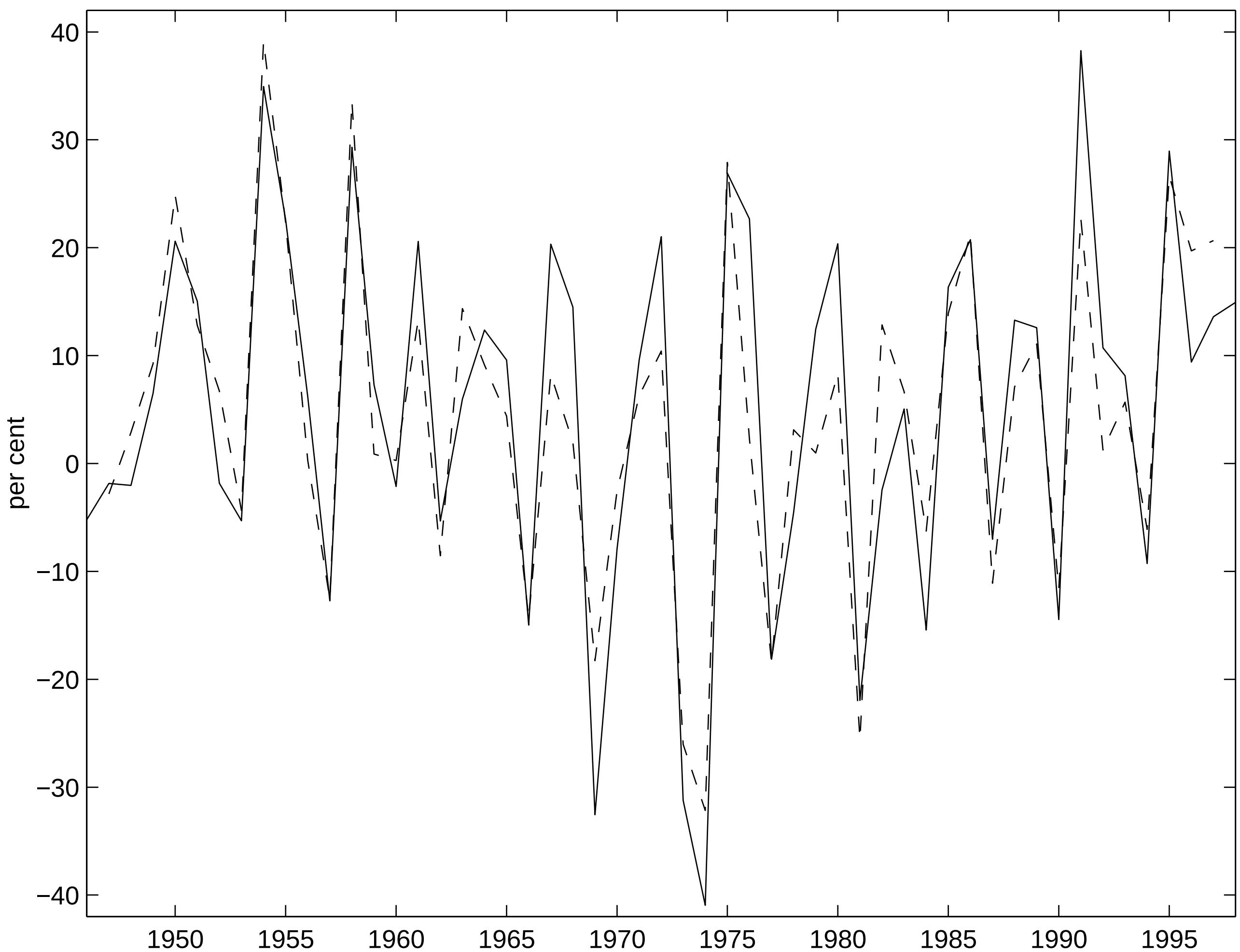

Figure 5: Empirical Excess Returns on Corporate Equity, and the S&P500 Index

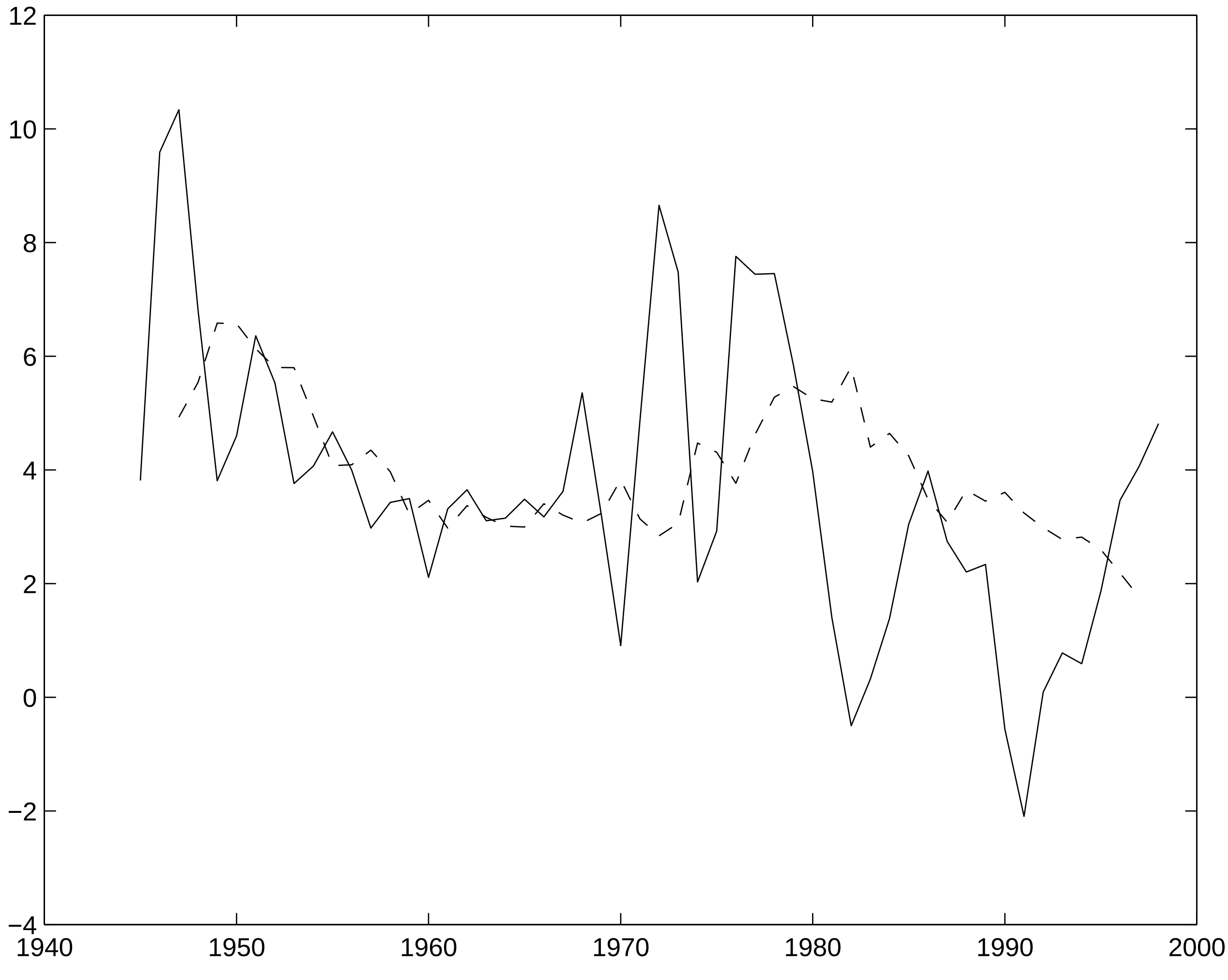

Figure 6: Kalman-Filter Estimate of the State Variable, and Empirical Dividend Yield on the S&P500 Index

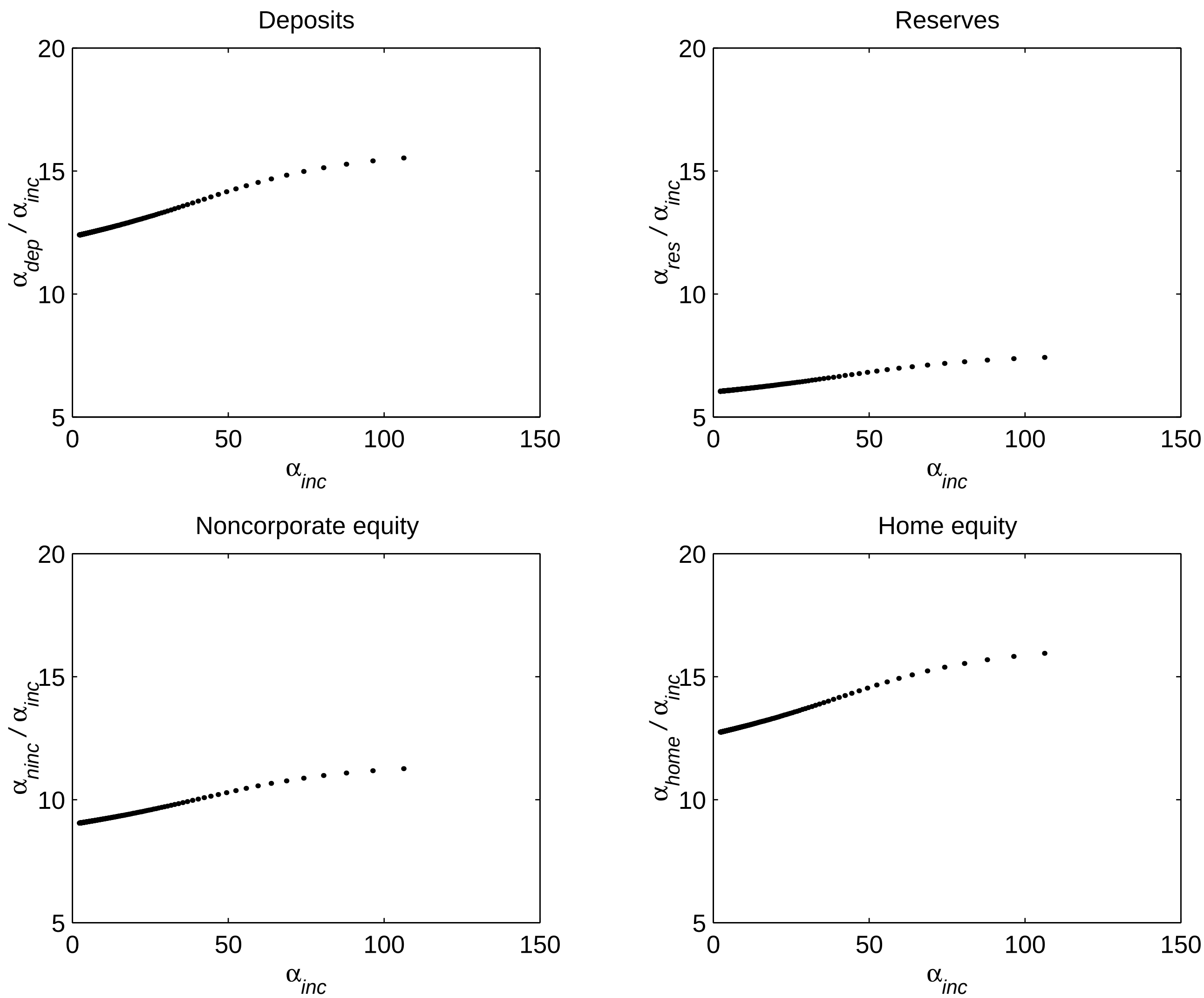

Figure 7: Predicted Composition of Risky Assets Relative to Corporate Equity